# Package ‘CUB’

February 23, 2024

**Title** A Class of Mixture Models for Ordinal Data

**Version** 1.1.5

**Description**
For ordinal rating data, estimate and test models within the family of CUB models and their extensions (where CUB stands for Combination of a discrete Uniform and a shifted Binomial distributions); Simulation routines, plotting facilities and fitting measures are also provided.

**Depends** R (>= 2.15.2), Formula

**License** GPL-2 | GPL-3

**Encoding** UTF-8

**Imports** methods

**LazyData** true

**RoxygenNote** 7.3.1

**NeedsCompilation** no

**Suggests** knitr, digest

**VignetteBuilder** knitr

**Repository** CRAN

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## R topics documented:

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`betabinomial` *Beta-Binomial probabilities of ordinal responses, with feeling and overdispersion parameters for each observation*

---

## Description

Compute the Beta-Binomial probabilities of ordinal responses, given feeling and overdispersion parameters for each observation.

## Usage

```
betabinomial(m,ordinal,csivett,phivett)
```

## Arguments

| | |
|---|---|
| `m` | Number of ordinal categories |
| `ordinal` | Vector of ordinal responses. Missing values are not allowed: they should be preliminarily deleted or imputed |
| `csivett` | Vector of feeling parameters of the Beta-Binomial distribution for given ordinal responses |
| `phivett` | Vector of overdispersion parameters of the Beta-Binomial distribution for given ordinal responses |

## Details

The Beta-Binomial distribution is the Binomial distribution in which the probability of success at each trial is random and follows the Beta distribution. It is frequently used in Bayesian statistics, empirical Bayes methods and classical statistics as an overdispersed binomial distribution.

## Value

A vector of the same length as ordinal, containing the Beta-Binomial probabilities of each observation, for the corresponding feeling and overdispersion parameters.

## References

Iannario, M. (2014). Modelling Uncertainty and Overdispersion in Ordinal Data, *Communications in Statistics - Theory and Methods*, **43**, 771–786
Piccolo D. (2015). Inferential issues for CUBE models with covariates. *Communications in Statistics - Theory and Methods*, **44**(23), 771–786.

## See Also

betar, betabinomialcsi

## Examples

```
data(relgoods)
m<-10
ordinal<-relgoods$Tv
age<-2014-relgoods$BirthYear
no_na<-na.omit(cbind(ordinal,age))
ordinal<-no_na[,1]; age<-no_na[,2]
lage<-log(age)-mean(log(age))
gama<-c(-0.6, -0.3)
csivett<-logis(lage,gama)
alpha<-c(-2.3,0.92);
ZZ<-cbind(1,lage)
phivett<-exp(ZZ%*%alpha)
pr<-betabinomial(m,ordinal,csivett,phivett)
plot(density(pr))
```

---

# betabinomialcsi — *Beta-Binomial probabilities of ordinal responses, given feeling parameter for each observation*

---

## Description

Compute the Beta-Binomial probabilities of given ordinal responses, with feeling parameter specified for each observation, and with the same overdispersion parameter for all the responses.

## Usage

```
betabinomialcsi(m,ordinal,csivett,phi)
```

## Arguments

| | |
|---|---|
| m | Number of ordinal categories |
| ordinal | Vector of ordinal responses. Missing values are not allowed: they should be preliminarily deleted or imputed |
| csivett | Vector of feeling parameters of the Beta-Binomial distribution for given ordinal responses |
| phi | Overdispersion parameter of the Beta-Binomial distribution |

## Value

A vector of the same length as ordinal: each entry is the Beta-Binomial probability for the given observation for the corresponding feeling and overdispersion parameters.

## References

Iannario, M. (2014). Modelling Uncertainty and Overdispersion in Ordinal Data, *Communications in Statistics - Theory and Methods*, **43**, 771–786
Piccolo D. (2015). Inferential issues for CUBE models with covariates. *Communications in Statistics - Theory and Methods*, **44**(23), 771–786.

## See Also

betar, betabinomial

## Examples

```
data(relgoods)
m<-10
ordinal<-relgoods$Tv
age<-2014-relgoods$BirthYear
no_na<-na.omit(cbind(ordinal,age))
ordinal<-no_na[,1]; age<-no_na[,2]
lage<-log(age)-mean(log(age))
gama<-c(-0.61,-0.31)
phi<-0.16
csivett<-logis(lage,gama)
pr<-betabinomialcsi(m,ordinal,csivett,phi)
plot(density(pr))
```

---

# betar *Beta-Binomial distribution*

---

## Description

Return the Beta-Binomial distribution with parameters $m$, $csi$ and $phi$.

## Usage

```
betar(m,csi,phi)
```

## Arguments

| | |
|---|---|
| m | Number of ordinal categories |
| csi | Feeling parameter of the Beta-Binomial distribution |
| phi | Overdispersion parameter of the Beta-Binomial distribution |

### Value

The vector of length $m$ of the Beta-Binomial distribution.

### References

Iannario, M. (2014). Modelling Uncertainty and Overdispersion in Ordinal Data, *Communications in Statistics - Theory and Methods*, **43**, 771–786

### See Also

betabinomial

### Examples

```
m<-9
csi<-0.8
phi<-0.2
pr<-betar(m,csi,phi)
plot(1:m,pr,type="h", main="Beta-Binomial distribution",xlab="Ordinal categories")
points(1:m,pr,pch=19)
```

---

## BIC.GEM *S3 BIC method for class "GEM"*

---

### Description

S3 BIC method for objects of class GEM.

### Usage

```
## S3 method for class 'GEM'
BIC(object, ...)
```

### Arguments

| | |
|---|---|
| object | An object of class "GEM" |
| ... | Other arguments |

### Value

BIC index for the fitted model.

### See Also

logLik, GEM

bitcsi *Shifted Binomial probabilities of ordinal responses*

## Description

Compute the shifted Binomial probabilities of ordinal responses.

## Usage

```
bitcsi(m,ordinal,csi)
```

## Arguments

| | |
|---|---|
| m | Number of ordinal categories |
| ordinal | Vector of ordinal responses |
| csi | Feeling parameter of the shifted Binomial distribution |

## Value

A vector of the same length as `ordinal`, where each entry is the shifted Binomial probability of the corresponding observation.

## References

Piccolo D. (2003). On the moments of a mixture of uniform and shifted binomial random variables, *Quaderni di Statistica*, **5**, 85–104

## See Also

probcub00, probcubp0, probcub0q

## Examples

```
data(univer)
m<-7
csi<-0.7
ordinal<-univer$informat
pr<-bitcsi(m,ordinal,csi)
```

`bitgama` *Shifted Binomial distribution with covariates*

## Description

Return the shifted Binomial probabilities of ordinal responses where the feeling component is explained by covariates via a logistic link.

## Usage

```
bitgama(m,ordinal,W,gama)
```

## Arguments

| | |
|---|---|
| `m` | Number of ordinal categories |
| `ordinal` | Vector of ordinal responses |
| `W` | Matrix of covariates for the feeling component |
| `gama` | Vector of parameters for the feeling component, with length equal to NCOL(W)+1 to account for an intercept term (first entry of `gama`) |

## Value

A vector of the same length as `ordinal`, where each entry is the shifted Binomial probability for the corresponding observation and feeling value.

## See Also

`logis`, `probcub0q`, `probcubpq`

## Examples

```
n<-100
m<-7
W<-sample(c(0,1),n,replace=TRUE)
gama<-c(0.2,-0.2)
csivett<-logis(W,gama)
ordinal<-rbinom(n,m-1,csivett)+1
pr<-bitgama(m,ordinal,W,gama)
```

chi2cub *Pearson $X^2$ statistic*

## Description

Compute the $X^2$ statistic of Pearson for CUB models with one or two discrete covariates for the feeling component.

## Usage

```
chi2cub(m,ordinal,W,pai,gama)
```

## Arguments

| | |
|---|---|
| `m` | Number of ordinal categories |
| `ordinal` | Vector of ordinal responses |
| `W` | Matrix of covariates for the feeling component |
| `pai` | Uncertainty parameter |
| `gama` | Vector of parameters for the feeling component, with length equal to NCOL(W)+1 to account for an intercept term (first entry of `gama`) |

## Details

No missing value should be present neither for `ordinal` nor for covariate matrices: thus, deletion or imputation procedures should be preliminarily run.

## Value

A list with the following components:

| | |
|---|---|
| `df` | Degrees of freedom |
| `chi2` | Value of the Pearson fitting measure |
| `dev` | Deviance indicator |

## References

Tutz, G. (2012). *Regression for Categorical Data*, Cambridge University Press, Cambridge

## Examples

```
data(univer)
m<-7
pai<-0.3
gama<-c(0.1,0.7)
ordinal<-univer$informat; W<-univer$gender;
pearson<-chi2cub(m,ordinal,W,pai,gama)
degfree<-pearson$df
statvalue<-pearson$chi2
deviance<-pearson$dev
```

`coef.GEM` *S3 Method: coef for class "GEM"*

## Description

S3 method: coef for objects of class `GEM`.

## Usage

```
## S3 method for class 'GEM'
coef(object, ...)
```

## Arguments

| | |
|---|---|
| `object` | An object of class `GEM` |
| `...` | Other arguments |

## Details

Returns estimated values of coefficients of the fitted model

## Value

ML estimates of parameters of the fitted GEM model.

## See Also

`GEM`, `summary`

`cormat` *Correlation matrix for estimated model*

## Description

Compute parameter correlation matrix for estimated model as returned by an object of class "GEM".

## Usage

```
cormat(object,digits=options()$digits)
```

## Arguments

| | |
|---|---|
| `object` | An object of class "GEM" |
| `digits` | Number of significant digits to be printed. Default is `options()$digits` |

### Value

Parameters correlation matrix for fitted GEM models.

### See Also

`GEM`, `vcov`

---

`cubevisual` *Plot an estimated CUBE model*

---

### Description

Plotting facility for the CUBE estimation of ordinal responses.

### Usage

```
cubevisual(ordinal,csiplot=FALSE,paiplot=FALSE,...)
```

### Arguments

| | |
|---|---|
| `ordinal` | Vector of ordinal responses |
| `csiplot` | Logical: should $\xi$ or $1-\xi$ be the $y$ coordinate |
| `paiplot` | Logical: should $\pi$ or $1-\pi$ be the $x$ coordinate |
| `...` | Additional arguments to be passed to `plot()` and `text()`. Optionally, the number `m` of ordinal categories may be passed: this is recommended if some category has zero frequency. |

### Details

It represents an estimated CUBE model as a point in the parameter space with the overdispersion being labeled.

### Value

For a CUBE model fitted to `ordinal`, by default it returns a plot of the estimated $(1-\pi, 1-\xi)$ as a point in the parameter space, labeled with the estimated overdispersion $\phi$. Depending on `csiplot` and `paiplot` and on desired output, $x$ and $y$ coordinates may be set to $\pi$ and $\xi$, respectively.

### Examples

```
data(univer)
ordinal<-univer$global
cubevisual(ordinal,xlim=c(0,0.5),main="Global Satisfaction",
   ylim=c(0.5,1),cex=0.8,digits=3,col="red")
```

`cubshevisual` *Plot an estimated CUB model with shelter*

## Description

Plotting facility for the CUB estimation of ordinal responses when a shelter effect is included

## Usage

```
cubshevisual(ordinal,shelter,csiplot=FALSE,paiplot=FALSE,...)
```

## Arguments

| | |
|---|---|
| `ordinal` | Vector of ordinal responses |
| `shelter` | Category corresponding to the shelter choice |
| `csiplot` | Logical: should $\xi$ or $1-\xi$ be the $y$ coordinate |
| `paiplot` | Logical: should $\pi$ or $1-\pi$ be the $x$ coordinate |
| `...` | Additional arguments to be passed to `plot()` and `text()`. Optionally, the number `m` of ordinal categories may be passed: this is recommended if some category has zero frequency. |

## Details

It represents an estimated CUB model with shelter effect as a point in the parameter space with shelter estimate indicated as label.

## Value

For a CUB model with shelter fitted to `ordinal`, by default it returns a plot of the estimated $(1-\pi, 1-\xi)$ as a point in the parameter space, labeled with the estimated shelter parameter $\delta$. Depending on `csiplot` and `paiplot` and on desired output, $x$ and $y$ coordinates may be set to $\pi$ and $\xi$, respectively.

## See Also

`cubvisual`, `multicub`

## Examples

```
data(univer)
ordinal<-univer$global
cubshevisual(ordinal,shelter=7,digits=3,col="blue",main="Global Satisfaction")
```

`cubvisual` *Plot an estimated CUB model*

### Description

Plotting facility for the CUB estimation of ordinal responses.

### Usage

```
cubvisual(ordinal,csiplot=FALSE,paiplot=FALSE,...)
```

### Arguments

| | |
|---|---|
| `ordinal` | Vector of ordinal responses |
| `csiplot` | Logical: should $\xi$ or $1-\xi$ be the $y$ coordinate |
| `paiplot` | Logical: should $\pi$ or $1-\pi$ be the $x$ coordinate |
| `...` | Additional arguments to be passed to `plot()` and `text()`. Optionally, the number `m` of ordinal categories may be passed: this is recommended if some category has zero frequency. |

### Details

It represents an estimated CUB model as a point in the parameter space with some useful options.

### Value

For a CUB model fit to `ordinal`, by default it returns a plot of the estimated $(1-\pi, 1-\xi)$ as a point in the parameter space. Depending on `csiplot` and `paiplot` and on desired output, $x$ and $y$ coordinates may be set to $\pi$ and $\xi$, respectively.

### Examples

```
data(univer)
ordinal<-univer$global
cubvisual(ordinal,xlim=c(0,0.5),ylim=c(0.5,1),cex=0.8,main="Global Satisfaction")
```

CUB_package *CUB package*

## Description

The analysis of human perceptions is often carried out by resorting to questionnaires, where respondents are asked to express ratings about the items being evaluated. The standard goal of the statistical framework proposed for this kind of data (e.g. cumulative models) is to explicitly characterize the respondents' perceptions about a latent trait, by taking into account, at the same time, the ordinal categorical scale of measurement of the involved statistical variables.
The new class of models starts from a particular assumption about the unconscious mechanism leading individuals' responses to choose an ordinal category on a rating scale. The basic idea derives from the awareness that two latent components move the psychological process of selection among discrete alternatives: attractiveness towards the item and uncertainty in the response. Both components of models concern the stochastic mechanism in term of feeling, which is an internal/personal movement of the subject towards the item, and uncertainty pertaining to the final choice.
Thus, on the basis of experimental data and statistical motivations, the response distribution is modelled as the convex Combination of a discrete Uniform and a shifted Binomial random variable (denoted as CUB model) whose parameters may be consistently estimated and validated by maximum likelihood inference. In addition, subjects' and objects' covariates can be included in the model in order to assess how the characteristics of the respondents may affect the ordinal score.
CUB models have been firstly introduced by Piccolo (2003) and implemented on real datasets concerning ratings and rankings by D'Elia and Piccolo (2005).
The CUB package allows the user to estimate and test CUB models and their extensions by using maximum likelihood methods: see Piccolo and Simone (2019a, 2019b) for an updated overview of methodological developments and applications. The accompanying vignettes supplies the user with detailed usage instructions and examples.
Acknowledgements: The Authors are grateful to Maria Antonietta Del Ferraro, Francesco Miranda and Giuseppe Porpora for their preliminary support in the implementation of the first version of the package.

## Details

| | |
|---|---|
| Package: | CUB |
| Type: | Package |
| Version: | 1.1.4 |
| Date: | 2017-10-11 |
| License: GPL-2 \| GPL-3 | |

## Author(s)

Maria Iannario, Domenico Piccolo, Rosaria Simone

## References

D'Elia A. (2003). Modelling ranks using the inverse hypergeometric distribution, *Statistical Modelling: an International Journal*, **3**, 65–78
Piccolo D. (2003). On the moments of a mixture of uniform and shifted binomial random variables, *Quaderni di Statistica*, **5**, 85–104
D'Elia A. and Piccolo D. (2005). A mixture model for preferences data analysis, *Computational Statistics & Data Analysis*, **49**, 917–937
Piccolo D. and Simone R. (2019a). The class of CUB models: statistical foundations, inferential issues and empirical evidence. *Statistical Methods and Applications*, **28**(3), 389–435.
Piccolo D. and Simone R. (2019b). Rejoinder to the discussions: The class of CUB models: statistical foundations, inferential issues and empirical evidence. *Statistical Methods and Applications*, **28**(3), 477-493.
Capecchi S. and Piccolo D. (2017). Dealing with heterogeneity in ordinal responses, *Quality and Quantity*, **51**(5), 2375–2393
*Metron*, **74**(2), 233–252.
Iannario M. and Piccolo D. (2016b). A generalized framework for modelling ordinal data. *Statistical Methods and Applications*, **25**, 163–189.

---

`deltaprob` *Mean difference of a discrete random variable*

---

## Description

Compute the Gini mean difference of a discrete distribution

## Usage

```
deltaprob(prob)
```

## Arguments

| | |
|---|---|
| `prob` | Vector of the probability distribution |

## Value

Numeric value of the Gini mean difference of the input probability distribution, computed according to the de Finetti-Paciello formulation.

## Examples

```
prob<-c(0.04,0.04,0.05,0.10,0.21,0.32,0.24)
deltaprob(prob)
```

---

`dissim` *Normalized dissimilarity measure*

---

## Description

Compute the normalized dissimilarity measure between observed relative frequencies and estimated (theoretical) probabilities of a discrete distribution.

## Usage

```
dissim(proba,probb)
```

## Arguments

| | |
|---|---|
| `proba` | Vector of observed relative frequencies |
| `probb` | Vector of estimated (theoretical) probabilities |

## Value

Numeric value of the dissimilarity index, assessing the distance to a perfect fit.

## Examples

```
proba<-c(0.01,0.03,0.08,0.07,0.27,0.37,0.17)
probb<-c(0.04,0.04,0.05,0.10,0.21,0.32,0.24)
dissim(proba,probb)
```

---

`ellecub` *Log-likelihood function of a CUB model without covariates*

---

## Description

Compute the log-likelihood function of a CUB model without covariates fitting ordinal responses, possibly with subjects' specific parameters.

## Usage

```
ellecub(m,ordinal,assepai,assecsi)
```

## Arguments

| | |
|---|---|
| `m` | Number of ordinal categories |
| `ordinal` | Vector of ordinal responses |
| `assepai` | Vector of uncertainty parameters for given observations (with the same length as `ordinal`) |
| `assecsi` | Vector of feeling parameters for given observations (with the same length as `ordinal`) |

## See Also

loglikCUB

## Examples

```
m<-7
n0<-230
n1<-270
bet<-c(-1.5,1.2)
gama<-c(0.5,-1.2)
pai0<-logis(0,bet); csi0<-logis(0,gama)
pai1<-logis(1,bet); csi1<-logis(1,gama)
ordinal0<-simcub(n0,m,pai0,csi0)
ordinal1<-simcub(n1,m,pai1,csi1)
ordinal<-c(ordinal0,ordinal1)
assepai<-c(rep(pai0,n0),rep(pai1,n1))
assecsi<-c(rep(csi0,n0),rep(csi1,n1))
lli<-ellecub(m,ordinal,assepai,assecsi)
```

---

expcub00 *Expectation of CUB distributions*

---

## Description

Compute the expectation of a CUB model without covariates.

## Usage

```
expcub00(m,pai,csi)
```

## Arguments

| | |
|---|---|
| m | Number of ordinal categories |
| pai | Uncertainty parameter |
| csi | Feeling parameter |

## References

Piccolo D. (2003). On the moments of a mixture of uniform and shifted binomial random variables. *Quaderni di Statistica*, **5**, 85–104

## See Also

varcub00, expcube, varcube

### Examples

```
m<-10
pai<-0.3
csi<-0.7
meancub<-expcub00(m,pai,csi)
```

---

## expcube *Expectation of CUBE models*

---

### Description

Compute the expectation of a CUBE model without covariates.

### Usage

```
expcube(m,pai,csi,phi)
```

### Arguments

| | |
|---|---|
| m | Number of ordinal categories |
| pai | Uncertainty parameter |
| csi | Feeling parameter |
| phi | Overdispersion parameter |

### References

Iannario M. (2014). Modelling Uncertainty and Overdispersion in Ordinal Data, *Communications in Statistics - Theory and Methods*, **43**, 771–786

Iannario, M. (2015). Detecting latent components in ordinal data with overdispersion by means of a mixture distribution, *Quality & Quantity*, **49**, 977–987

### See Also

varcube, varcub00, expcub00

### Examples

```
m<-10
pai<-0.1
csi<-0.7
phi<-0.2
meancube<-expcube(m,pai,csi,phi)
```

## fitted.GEM — *S3 method "fitted" for class "GEM"*

### Description

S3 method fitted for objects of class GEM.

### Usage

```
## S3 method for class 'GEM'
fitted(object, ...)
```

### Arguments

| | |
|---|---|
| object | An object of class GEM |
| ... | Other arguments |

### Details

Returns the fitted probability distribution for GEM models with no covariates. If only one dichotomous covariate is included in the model to explain some components, it returns the fitted probability distribution for each profile.

### See Also

GEM

### Examples

```
fitcub<-GEM(Formula(global~0|freqserv|0),family="cub",data=univer)
fitted(fitcub,digits=4)
```

## GEM — *Main function for GEM models*

### Description

Main function to estimate and validate GEneralized Mixture models with uncertainty.

### Usage

```
GEM(Formula,family=c("cub","cube","ihg","cush"),data,...)
```

### Arguments

| | |
|---|---|
| `Formula` | Object of class Formula. Response variable is the vector of ordinal observations - see Details. |
| `family` | Character string indicating which class of GEM models to fit. |
| `data` | an optional data frame (or object coercible by `as.data.frame` to a data frame) containing the variables in the model. If missing, the variables are taken from `environment(Formula)`. |
| `...` | Additional arguments to be passed for the specification of the model. See details and examples. |

### Details

It is the main function for GEM models estimation, calling for the corresponding function for the specified subclass. The number of categories `m` is internally retrieved but it is advisable to pass it as an argument to the call if some category has zero frequency.
If `family="cub"`, then a CUB mixture model is fitted to the data to explain uncertainty, feeling and possible shelter effect by further passing the extra argument `shelter` for the corresponding category. Subjects' covariates can be included by specifying covariates matrices in the Formula as `ordinal~Y|W|X`, to explain uncertainty (Y), feeling (W) or shelter (X). Notice that covariates for shelter effect can be included only if specified for both feeling and uncertaint (GeCUB models).
If `family="cube"`, then a CUBE mixture model (Combination of Uniform and Beta-Binomial) is fitted to the data to explain uncertainty, feeling and overdispersion. Subjects' covariates can be also included to explain the feeling component or all the three components by specifying covariates matrices in the Formula as `ordinal~Y|W|Z` to explain uncertainty (Y), feeling (W) or overdispersion (Z). An extra logical argument `expinform` indicates whether or not to use the expected or the observed information matrix (default is FALSE).
If `family="ihg"`, then an IHG model is fitted to the data. IHG models (Inverse Hypergeometric) are nested into CUBE models (see the references below). The parameter $\theta$ gives the probability of observing the first category and is therefore a direct measure of preference, attraction, pleasantness toward the investigated item. This is the reason why $\theta$ is customarily referred to as the preference parameter of the IHG model. Covariates for the preference parameter $\theta$ have to be specified in matrix form in the Formula as `ordinal~U`.
If `family="cush"`, then a CUSH model is fitted to the data (Combination of Uniform and SHelter effect). The category corresponding to the inflation should be passed via argument `shelter`. Covariates for the shelter parameter $\delta$ are specified in matrix form Formula as `ordinal~X`.
Even if no covariate is included in the model for a given component, the corresponding model matrix needs always to be specified: in this case, it should be set to 0 (see examples below). Extra arguments include the maximum number of iterations (`maxiter`, default: `maxiter=500`) for the optimization algorithm and the required error tolerance (`toler`, default: `toler=1e-6`).
Standard methods: `logLik()`, `BIC()`, `vcov()`, `fitted()`, `coef()`, `print()`, `summary()` are implemented.
The optimization procedure is run via `optim()` when required. If the estimated variance-covariance matrix is not positive definite, the function returns a warning message and produces a matrix with NA entries.

### Value

An object of the class "GEM" is a list containing the following elements:

| | |
|---|---|
| `estimates` | Maximum likelihood estimates of parameters |
| `loglik` | Log-likelihood function at the final estimates |
| `varmat` | Variance-covariance matrix of final estimates |
| `niter` | Number of executed iterations |
| `BIC` | BIC index for the estimated model |
| `ordinal` | Vector of ordinal responses on which the model has been fitted |
| `time` | Processor time for execution |
| `ellipsis` | Retrieve the arguments passed to the call and extra arguments generated via the call |
| `family` | Character string indicating the sub-class of the fitted model |
| `formula` | Returns the Formula of the call for the fitted model |
| `call` | Returns the executed call |

## References

D'Elia A. (2003). Modelling ranks using the inverse hypergeometric distribution, *Statistical Modelling: an International Journal*, **3**, 65–78
D'Elia A. and Piccolo D. (2005). A mixture model for preferences data analysis, *Computational Statistics & Data Analysis*, **49**, 917–937
Capecchi S. and Piccolo D. (2017). Dealing with heterogeneity in ordinal responses, *Quality and Quantity*, **51**(5), 2375–2393
Iannario M. (2014). Modelling Uncertainty and Overdispersion in Ordinal Data, *Communications in Statistics - Theory and Methods*, **43**, 771–786
Piccolo D. (2015). Inferential issues for CUBE models with covariates, *Communications in Statistics. Theory and Methods*, **44**(23), 771–786.
Iannario M. (2015). Detecting latent components in ordinal data with overdispersion by means of a mixture distribution, *Quality & Quantity*, **49**, 977–987
Iannario M. and Piccolo D. (2016a). A comprehensive framework for regression models of ordinal data. *Metron*, **74**(2), 233–252.
Iannario M. and Piccolo D. (2016b). A generalized framework for modelling ordinal data. *Statistical Methods and Applications*, **25**, 163–189.

## See Also

`logLik`, `coef`, `BIC`, `makeplot`, `summary`, `vcov`, `fitted`, `cormat`

## Examples

```
library(CUB)
## CUB models with no covariates
model<-GEM(Formula(Walking~0|0|0),family="cub",data=relgoods)
coef(model,digits=5)     # Estimated parameter vector (pai,csi)
logLik(model)            # Log-likelihood function at ML estimates
vcov(model,digits=4)     # Estimated Variance-Covariance matrix
cormat(model)            # Parameter Correlation matrix
```

```
fitted(model)            # Fitted probability distribution
makeplot(model)
################
## CUB model with shelter effect
model<-GEM(Formula(officeho~0|0|0),family="cub",shelter=7,data=univer)
BICshe<-BIC(model,digits=4)
################
## CUB model with covariate for uncertainty
modelcovpai<-GEM(Formula(Parents~Smoking|0|0),family="cub",data=relgoods)
fitted(modelcovpai)
makeplot(modelcovpai)
################
## CUB model with covariates for both uncertainty and feeling components
data(univer)
model<-GEM(Formula(global~gender|freqserv|0),family="cub",data=univer,maxiter=50,toler=1e-2)
param<-coef(model)
bet<-param[1:2]      # ML estimates of coefficients for uncertainty covariate: gender
gama<-param[3:4]     # ML estimates of coefficients for feeling covariate: lage
##################
## CUBE models with no covariates
model<-GEM(Formula(MeetRelatives~0|0|0),family="cube",starting=c(0.5,0.5,0.1),
  data=relgoods,expinform=TRUE,maxiter=50,toler=1e-2)
coef(model,digits=4)       # Final ML estimates
vcov(model)
fitted(model)
makeplot(model)
summary(model)
##################
## IHG with covariates
modelcov<-GEM(willingn~freqserv,family="ihg",data=univer)
omega<-coef(modelcov)      ## ML estimates
maxlik<-logLik(modelcov)   ##
makeplot(modelcov)
summary(modelcov)
###################
## CUSH models without covariate
model<-GEM(Dog~0,family="cush",shelter=1,data=relgoods)
delta<-coef(model)      # ML estimates of delta
maxlik<-logLik(model)   # Log-likelihood at ML estimates
summary(model)
makeplot(model)
```

---

`gini` *Normalized Gini heterogeneity index*

---

## Description

Compute the normalized Gini heterogeneity index for a given discrete probability distribution.

## Usage

```
gini(prob)
```

## Arguments

| | |
|---|---|
| `prob` | Vector of probability distribution or relative frequencies |

## See Also

`laakso`

## Examples

```
prob<-c(0.04,0.04,0.05,0.10,0.21,0.32,0.24)
gini(prob)
```

---

# `inibest` *Preliminary estimators for CUB models without covariates*

---

## Description

Compute preliminary parameter estimates of a CUB model without covariates for given ordinal responses. These preliminary estimators are used within the package code to start the E-M algorithm.

## Usage

```
inibest(m,freq)
```

## Arguments

| | |
|---|---|
| `m` | Number of ordinal categories |
| `freq` | Vector of the absolute frequencies of given ordinal responses |

## Value

A vector $(\pi, \xi)$ of the initial parameter estimates for a CUB model without covariates, given the absolute frequency distribution of ordinal responses

## References

Iannario M. (2009). A comparison of preliminary estimators in a class of ordinal data models, *Statistica & Applicazioni*, **VII**, 25–44
Iannario M. (2012). Preliminary estimators for a mixture model of ordinal data, *Advances in Data Analysis and Classification*, **6**, 163–184

## See Also

`inibestgama`

## Examples

```
m<-9
freq<-c(10,24,28,36,50,43,23,12,5)
estim<-inibest(m,freq)
pai<-estim[1]
csi<-estim[2]
```

---

inibestcube *Naive estimates for CUBE models without covariates*

---

## Description

Compute *naive* parameter estimates of a CUBE model without covariates for given ordinal responses. These preliminary estimators are used within the package code to start the E-M algorithm.

## Usage

```
inibestcube(m,ordinal)
```

## Arguments

| | |
|---|---|
| m | Number of ordinal categories |
| ordinal | Vector of ordinal responses |

## Value

A vector $(\pi, \xi, \phi)$ of parameter estimates of a CUBE model without covariates.

## See Also

inibestcubecov, inibestcubecsi

## Examples

```
data(relgoods)
m<-10
ordinal<-relgoods$SocialNetwork
estim<-inibestcube(m,ordinal)     # Preliminary estimates (pai,csi,phi)
```

`inibestcubecov` *Preliminary parameter estimates for CUBE models with covariates*

### Description

Compute preliminary parameter estimates for a CUBE model with covariates for all the three parameters. These estimates are set as initial values to start the E-M algorithm within maximum likelihood estimation.

### Usage

```
inibestcubecov(m,ordinal,Y,W,Z)
```

### Arguments

| | |
|---|---|
| `m` | Number of ordinal categories |
| `ordinal` | Vector of ordinal responses |
| `Y` | Matrix of selected covariates to explain the uncertainty parameter |
| `W` | Matrix of selected covariates to explain the feeling parameter |
| `Z` | Matrix of selected covariates to explain the overdispersion parameter |

### Value

A vector `(inibet, inigama, inialpha)` of preliminary estimates of parameter vectors for $\pi = \pi(\boldsymbol{\beta})$, $\xi = \xi(\boldsymbol{\gamma})$, $\phi = \phi(\boldsymbol{\alpha})$, respectively, of a CUBE model with covariates for all the three parameters. In details, `inibet`, `inigama` and `inialpha` have length equal to NCOL(Y)+1, NCOL(W)+1 and NCOL(Z)+1, respectively, to account for an intercept term for each component.

### See Also

`inibestcube`, `inibestcubecsi`, `inibestgama`

### Examples

```
data(relgoods)
m<-10
naord<-which(is.na(relgoods$Tv))
nacovpai<-which(is.na(relgoods$Gender))
nacovcsi<-which(is.na(relgoods$year.12))
nacovphi<-which(is.na(relgoods$EducationDegree))
na<-union(union(naord,nacovpai),union(nacovcsi,nacovphi))
ordinal<-relgoods$Tv[-na]
Y<-relgoods$Gender[-na]
W<-relgoods$year.12[-na]
Z<-relgoods$EducationDegree[-na]
ini<-inibestcubecov(m,ordinal,Y,W,Z)
p<-NCOL(Y)
```

```
q<-NCOL(W)
inibet<-ini[1:(p+1)]              # Preliminary estimates for uncertainty
inigama<-ini[(p+2):(p+q+2)]       # Preliminary estimates for feeling
inialpha<-ini[(p+q+3):length(ini)] # Preliminary estimates for overdispersion
```

---

inibestcubecsi *Preliminary estimates of parameters for CUBE models with covariates only for feeling*

---

### Description

Compute preliminary parameter estimates of a CUBE model with covariates only for feeling, given ordinal responses. These estimates are set as initial values to start the corresponding E-M algorithm within the package.

### Usage

```
inibestcubecsi(m,ordinal,W,starting,maxiter,toler)
```

### Arguments

| | |
|---|---|
| m | Number of ordinal categories |
| ordinal | Vector of ordinal responses |
| W | Matrix of selected covariates to explain the feeling component |
| starting | Starting values for preliminary estimation of a CUBE without covariate |
| maxiter | Maximum number of iterations allowed for preliminary iterations |
| toler | Fixed error tolerance for final estimates for preliminary iterations |

### Details

Preliminary estimates for the uncertainty and the overdispersion parameters are computed by short runs of EM. As to the feeling component, it considers the nested CUB model with covariates and calls `inibestgama` to derive initial estimates for the coefficients of the selected covariates for feeling.

### Value

A vector `(pai, gamaest, phi)`, where `pai` is the initial estimate for the uncertainty parameter, `gamaest` is the vector of initial estimates for the feeling component (including an intercept term in the first entry), and `phi` is the initial estimate for the overdispersion parameter.

### See Also

`inibestcube`, `inibestcubecov`, `inibestgama`

## Examples

```
data(relgoods)
isnacov<-which(is.na(relgoods$Gender))
isnaord<-which(is.na(relgoods$Tv))
na<-union(isnacov,isnaord)
ordinal<-relgoods$Tv[-na]; W<-relgoods$Gender[-na]
m<-10
starting<-rep(0.1,3)
ini<-inibestcubecsi(m,ordinal,W,starting,maxiter=100,toler=1e-3)
nparam<-length(ini)
pai<-ini[1]                 # Preliminary estimates for uncertainty component
gamaest<-ini[2:(nparam-1)]  # Preliminary estimates for coefficients of feeling covariates
phi<-ini[nparam]            # Preliminary estimates for overdispersion component
```

---

# inibestgama — *Preliminary parameter estimates of a CUB model with covariates for feeling*

---

## Description

Compute preliminary parameter estimates for the feeling component of a CUB model fitted to ordinal responses These estimates are set as initial values for parameters to start the E-M algorithm.

## Usage

```
inibestgama(m,ordinal,W)
```

## Arguments

| | |
|---|---|
| `m` | Number of ordinal categories |
| `ordinal` | Vector of ordinal responses |
| `W` | Matrix of selected covariates for explaining the feeling component |

## Value

A vector of length equal to NCOL(W)+1, whose entries are the preliminary estimates of the parameters for the feeling component, including an intercept term as first entry.

## References

Iannario M. (2008). Selecting feeling covariates in rating surveys, *Rivista di Statistica Applicata*, **20**, 103–116
Iannario M. (2009). A comparison of preliminary estimators in a class of ordinal data models, *Statistica & Applicazioni*, **VII**, 25–44
Iannario M. (2012). Preliminary estimators for a mixture model of ordinal data, *Advances in Data Analysis and Classification*, **6**, 163–184

## See Also

inibest, inibestcubecsi

## Examples

```
data(univer)
m<-7; ordinal<-univer$global; cov<-univer$diploma
ini<-inibestgama(m,ordinal,W=cov)
```

---

inigrid *Grid-based preliminary parameter estimates for CUB models*

---

## Description

Compute the log-likelihood function of a CUB model with parameter vector $(\pi, \xi)$ ranging in the Cartesian product between $x$ and $y$, for a given absolute frequency distribution.

## Usage

```
inigrid(m,freq,x,y)
```

## Arguments

| | |
|---|---|
| m | Number of ordinal categories |
| freq | Vector of length $m$ of the absolute frequency distribution |
| x | A set of values to assign to the uncertainty parameter $\pi$ |
| y | A set of values to assign to the feeling parameter $\xi$ |

## Value

It returns the parameter vector corresponding to the maximum value of the log-likelihood for a CUB model without covariates for given frequencies.

## See Also

inibest

## Examples

```
m<-9
x<-c(0.1,0.4,0.6,0.8)
y<-c(0.2, 0.5,0.7)
freq<-c(10,24,28,36,50,43,23,12,5)
ini<-inigrid(m,freq,x,y)
pai<-ini[1]
csi<-ini[2]
```

`iniihg` *Moment estimate for the preference parameter of the IHG distribution*

## Description

Compute the moment estimate of the preference parameter of the IHG distribution. This preliminary estimate is set as initial value within the optimization procedure for an IHG model fitting the observed frequencies.

## Usage

```
iniihg(m,freq)
```

## Arguments

| | |
|---|---|
| `m` | Number of ordinal categories |
| `freq` | Vector of the absolute frequency distribution of the categories |

## Value

Moment estimator of the preference parameter $\theta$.

## References

D'Elia A. (2003). Modelling ranks using the inverse hypergeometric distribution, *Statistical Modelling: an International Journal*, **3**, 65–78.

## See Also

`inibest`, `inibestcube`

## Examples

```
m<-9
freq<-c(70,51,48,38,29,23,12,10,5)
initheta<-iniihg(m,freq)
```

`laakso` *Normalized Laakso and Taagepera heterogeneity index*

## Description

Compute the normalized Laakso and Taagepera heterogeneity index for a given discrete probability distribution.

## Usage

```
laakso(prob)
```

## Arguments

| | |
|---|---|
| `prob` | Vector of a probability or relative frequency distribution |

## References

Laakso, M. and Taagepera, R. (1989). Effective number of parties: a measure with application to West Europe, *Comparative Political Studies*, **12**, 3–27.

## See Also

`gini`

## Examples

```
prob<-c(0.04,0.04,0.05,0.10,0.21,0.32,0.24)
laakso(prob)
```

`logis` *The logistic transform*

## Description

Create a matrix YY binding array `Y` with a vector of ones, placed as the first column of YY. It applies the logistic transform componentwise to the standard matrix multiplication between YY and `param`.

## Usage

```
logis(Y,param)
```

## Arguments

| | |
|---|---|
| `Y` | A generic matrix or one dimensional array |
| `param` | Vector of coefficients, whose length is NCOL(Y) + 1 (to consider also an intercept term) |

### Value

Return a vector whose length is NROW(Y) and whose i-th component is the logistic function at the scalar product between the i-th row of YY and the vector `param`.

### Examples

```
n<-50
Y<-sample(c(1,2,3),n,replace=TRUE)
param<-c(0.2,0.7)
logis(Y,param)
```

---

## logLik.GEM *logLik S3 Method for class "GEM"*

---

### Description

S3 method: logLik() for objects of class "GEM".

### Usage

```
## S3 method for class 'GEM'
logLik(object, ...)
```

### Arguments

| | |
|---|---|
| `object` | An object of class "GEM" |
| `...` | Other arguments |

### Value

Log-likelihood at the final ML estimates for parameters of the fitted GEM model.

### See Also

`loglikCUB`, `loglikCUBE`, `GEM`, `loglikIHG`, `loglikCUSH`, `BIC` loglikCUB *Log-likelihood function for CUB models*

## Description

Compute the log-likelihood value of a CUB model fitting given data, with or without covariates to explain the feeling and uncertainty components, or for extended CUB models with shelter effect.

## Usage

```
loglikCUB(ordinal,m,param,Y=0,W=0,X=0,shelter=0)
```

## Arguments

| | |
|---|---|
| `ordinal` | Vector of ordinal responses |
| `m` | Number of ordinal categories |
| `param` | Vector of parameters for the specified CUB model |
| `Y` | Matrix of selected covariates to explain the uncertainty component (default: no covariate is included in the model) |
| `W` | Matrix of selected covariates to explain the feeling component (default: no covariate is included in the model) |
| `X` | Matrix of selected covariates to explain the shelter effect (default: no covariate is included in the model) |
| `shelter` | Category corresponding to the shelter choice (default: no shelter effect is included in the model) |

## Details

If no covariate is included in the model, then `param` should be given in the form $(\pi, \xi)$. More generally, it should have the form $(\boldsymbol{\beta}, \boldsymbol{\gamma})$ where, respectively, $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ are the vectors of coefficients explaining the uncertainty and the feeling components, with length NCOL(Y)+1 and NCOL(W)+1 to account for an intercept term in the first entry. When shelter effect is considered, `param` corresponds to the first possibile parameterization and hence should be given as `(pai1,pai2,csi)`. No missing value should be present neither for `ordinal` nor for covariate matrices: thus, deletion or imputation procedures should be preliminarily run.

## See Also

`logLik`

## Examples

```
## Log-likelihood of a CUB model with no covariate
m<-9; n<-300
pai<-0.6; csi<-0.4
ordinal<-simcub(n,m,pai,csi)
param<-c(pai,csi)
loglikcub<-loglikCUB(ordinal,m,param)
##################################
## Log-likelihood of a CUB model with covariate for uncertainty
data(relgoods)
m<-10
naord<-which(is.na(relgoods$Physician))
nacov<-which(is.na(relgoods$Gender))
na<-union(naord,nacov)
ordinal<-relgoods$Physician[-na]; Y<-relgoods$Gender[-na]
bbet<-c(-0.81,0.93); ccsi<-0.2
param<-c(bbet,ccsi)
loglikcubp0<-loglikCUB(ordinal,m,param,Y=Y)
#######################
## Log-likelihood of a CUB model with covariate for feeling
data(relgoods)
m<-10
naord<-which(is.na(relgoods$Physician))
nacov<-which(is.na(relgoods$Gender))
na<-union(naord,nacov)
ordinal<-relgoods$Physician[-na]; W<-relgoods$Gender[-na]
pai<-0.44; gama<-c(-0.91,-0.7)
param<-c(pai,gama)
loglikcub0q<-loglikCUB(ordinal,m,param,W=W)
#######################
## Log-likelihood of a CUB model with covariates for both parameters
data(relgoods)
m<-10
naord<-which(is.na(relgoods$Walking))
nacovpai<-which(is.na(relgoods$Gender))
nacovcsi<-which(is.na(relgoods$Smoking))
na<-union(naord,union(nacovpai,nacovcsi))
ordinal<-relgoods$Walking[-na]
Y<-relgoods$Gender[-na]; W<-relgoods$Smoking[-na]
bet<-c(-0.45,-0.48); gama<-c(-0.55,-0.43)
param<-c(bet,gama)
loglikcubpq<-loglikCUB(ordinal,m,param,Y=Y,W=W)
#######################
### Log-likelihood of a CUB model with shelter effect
m<-7; n<-400
pai<-0.7; csi<-0.16; delta<-0.15
shelter<-5
ordinal<-simcubshe(n,m,pai,csi,delta,shelter)
pai1<- pai*(1-delta); pai2<-1-pai1-delta
param<-c(pai1,pai2,csi)
loglik<-loglikCUB(ordinal,m,param,shelter=shelter)
```

`loglikCUBE` *Log-likelihood function for CUBE models*

## Description

Compute the log-likelihood function for CUBE models. It is possible to include covariates in the model for explaining the feeling component or all the three parameters.

## Usage

```
loglikCUBE(ordinal,m,param,Y=0,W=0,Z=0)
```

## Arguments

| | |
|---|---|
| `ordinal` | Vector of ordinal responses |
| `m` | Number of ordinal categories |
| `param` | Vector of parameters for the specified CUBE model |
| `Y` | Matrix of selected covariates to explain the uncertainty component (default: no covariate is included in the model) |
| `W` | Matrix of selected covariates to explain the feeling component (default: no covariate is included in the model) |
| `Z` | Matrix of selected covariates to explain the overdispersion component (default: no covariate is included in the model) |

## Details

If no covariate is included in the model, then `param` has the form $(\pi, \xi, \phi)$. More generally, it has the form $(\boldsymbol{\beta}, \boldsymbol{\gamma}, \boldsymbol{\alpha})$ where, respectively, $\boldsymbol{\beta}$,$\boldsymbol{\gamma}$, $\boldsymbol{\alpha}$ are the vectors of coefficients explaining the uncertainty, the feeling and the overdispersion components, with length NCOL(Y)+1, NCOL(W)+1, NCOL(Z)+1 to account for an intercept term in the first entry. No missing value should be present neither for `ordinal` nor for covariate matrices: thus, deletion or imputation procedures should be preliminarily run.

## See Also

`logLik`

## Examples

```
#### Log-likelihood of a CUBE model with no covariate
m<-7; n<-400
pai<-0.83; csi<-0.19; phi<-0.045
ordinal<-simcube(n,m,pai,csi,phi)
loglik<-loglikCUBE(ordinal,m,param=c(pai,csi,phi))
##################################
#### Log-likelihood of a CUBE model with covariate for feeling
data(relgoods)
```

```
m<-10
nacov<-which(is.na(relgoods$BirthYear))
naord<-which(is.na(relgoods$Tv))
na<-union(nacov,naord)
age<-2014-relgoods$BirthYear[-na]
lage<-log(age)-mean(log(age))
ordinal<-relgoods$Tv[-na]; W<-lage
pai<-0.63; gama<-c(-0.61,-0.31); phi<-0.16
param<-c(pai,gama,phi)
loglik<-loglikCUBE(ordinal,m,param,W=W)
########## Log-likelihood of a CUBE model with covariates for all parameters
Y<-W<-Z<-lage
bet<-c(0.18, 1.03); gama<-c(-0.6, -0.3); alpha<-c(-2.3,0.92)
param<-c(bet,gama,alpha)
loglik<-loglikCUBE(ordinal,m,param,Y=Y,W=W,Z=Z)
```

---

`loglikCUSH` *Log-likelihood function for CUSH models*

---

### Description

Compute the log-likelihood function for CUSH models with or without covariates to explain the shelter effect.

### Usage

```
loglikCUSH(ordinal,m,param,shelter,X=0)
```

### Arguments

| | |
|---|---|
| `ordinal` | Vector of ordinal responses |
| `m` | Number of ordinal categories |
| `param` | Vector of parameters for the specified CUSH model |
| `shelter` | Category corresponding to the shelter choice |
| `X` | Matrix of selected covariates to explain the shelter effect (default: no covariate is included in the model) |

### Details

If no covariate is included in the model, then `param` is the estimate of the shelter parameter (delta), otherwise `param` has length equal to NCOL(X) + 1 to account for an intercept term (first entry). No missing value should be present neither for `ordinal` nor for `X`.

### See Also

`GEM`, `logLik`

### Examples

```
## Log-likelihood of CUSH model without covariates
n<-300
m<-7
shelter<-2; delta<-0.4
ordinal<-simcush(n,m,delta,shelter)
loglik<-loglikCUSH(ordinal,m,param=delta,shelter)
#####################
## Log-likelihood of CUSH model with covariates
data(relgoods)
m<-10
naord<-which(is.na(relgoods$SocialNetwork))
nacov<-which(is.na(relgoods$Gender))
na<-union(nacov,naord)
ordinal<-relgoods$SocialNetwork[-na]; cov<-relgoods$Gender[-na]
omega<-c(-2.29, 0.62)
loglikcov<-loglikCUSH(ordinal,m,param=omega,shelter=1,X=cov)
```

---

## `loglikIHG` *Log-likelihood function for IHG models*

---

### Description

Compute the log-likelihood function for IHG models with or without covariates to explain the preference parameter.

### Usage

```
loglikIHG(ordinal,m,param,U=0)
```

### Arguments

| | |
|---|---|
| `ordinal` | Vector of ordinal responses |
| `m` | Number of ordinal categories |
| `param` | Vector of parameters for the specified IHG model |
| `U` | Matrix of selected covariates to explain the preference parameter (default: no covariate is included in the model) |

### Details

If no covariate is included in the model, then `param` is the estimate of the preference parameter ($theta$), otherwise `param` has length equal to NCOL(U) + 1 to account for an intercept term (first entry). No missing value should be present neither for `ordinal` nor for `U`.

### See Also

`GEM`, `logLik`

### Examples

```
#### Log-likelihood of an IHG model with no covariate
m<-10; theta<-0.14; n<-300
ordinal<-simihg(n,m,theta)
loglik<-loglikIHG(ordinal,m,param=theta)
##################################
#### Log-likelihood of a IHG model with covariate
data(relgoods)
m<-10
naord<-which(is.na(relgoods$HandWork))
nacov<-which(is.na(relgoods$Gender))
na<-union(naord,nacov)
ordinal<-relgoods$HandWork[-na]; U<-relgoods$Gender[-na]
nu<-c(-1.55,-0.11)     # first entry: intercept term
loglik<-loglikIHG(ordinal,m,param=nu,U=U); loglik
```

---

## `logscore` *Logarithmic score*

---

### Description

Compute the logarithmic score of a CUB model with covariates both for the uncertainty and the feeling parameters.

### Usage

```
logscore(m,ordinal,Y,W,bet,gama)
```

### Arguments

| | |
|---|---|
| `m` | Number of ordinal categories |
| `ordinal` | Vector of ordinal responses |
| `Y` | Matrix of covariates for explaining the uncertainty component |
| `W` | Matrix of covariates for explaining the feeling component |
| `bet` | Vector of parameters for the uncertainty component, with length NCOL(Y)+1 to account for an intercept term (first entry of `bet`) |
| `gama` | Vector of parameters for the feeling component, with length NCOL(W)+1 to account for an intercept term (first entry of `gama`) |

### Details

No missing value should be present neither for `ordinal` nor for covariate matrices: thus, deletion or imputation procedures should be preliminarily run.

### References

Tutz, G. (2012). *Regression for Categorical Data*, Cambridge University Press, Cambridge

### Examples

```
data(relgoods)
m<-10
naord<-which(is.na(relgoods$Walking))
nacovpai<-which(is.na(relgoods$Gender))
nacovcsi<-which(is.na(relgoods$Smoking))
na<-union(naord,union(nacovpai,nacovcsi))
ordinal<-relgoods$Walking[-na]
Y<-relgoods$Gender[-na]
W<-relgoods$Smoking[-na]
bet<-c(-0.45,-0.48)
gama<-c(-0.55,-0.43)
logscore(m,ordinal,Y=Y,W=W,bet,gama)
```

---

## makeplot — *Plot facilities for GEM objects*

---

### Description

Plot facilities for objects of class "GEM".

### Usage

```
makeplot(object)
```

### Arguments

| | |
|---|---|
| `object` | An object of class "GEM" |

### Details

Returns a plot comparing fitted probabilities and observed relative frequencies for GEM models without covariates. If only one explanatory dichotomous variable is included in the model for one or all components, then the function returns a plot comparing the distributions of the responses conditioned to the value of the covariate.

### See Also

`cubvisual`, `cubevisual`, `cubshevisual`, `multicub`, `multicube` `multicub` *Joint plot of estimated CUB models in the parameter space*

## Description

Return a plot of estimated CUB models represented as points in the parameter space.

## Usage

```
multicub(listord,mvett,csiplot=FALSE,paiplot=FALSE,...)
```

## Arguments

| | |
|---|---|
| `listord` | A data matrix, data frame, or list of vectors of ordinal observations (for variables with different number of observations) |
| `mvett` | Vector of number of categories for ordinal variables in `listord` (optional: if missing, the number of categories is retrieved from data: it is advisable to specify it in case some category has zero frequency) |
| `csiplot` | Logical: should $\xi$ or $1-\xi$ be the $y$ coordinate |
| `paiplot` | Logical: should $\pi$ or $1-\pi$ be the $x$ coordinate |
| `...` | Additional arguments to be passed to `plot`, `text`, and `GEM` |

## Value

Fit a CUB model to list elements, and then by default it returns a plot of the estimated $(1-\pi, 1-\xi)$ as points in the parameter space. Depending on `csiplot` and `paiplot` and on desired output, $x$ and $y$ coordinates may be set to $\pi$ and $\xi$, respectively.

## Examples

```
data(univer)
listord<-univer[,8:12]
multicub(listord,colours=rep("red",5),cex=c(0.4,0.6,0.8,1,1.2),
  pch=c(1,2,3,4,5),xlim=c(0,0.4),ylim=c(0.75,1),pos=c(1,3,3,3,3))
###############################
m1<-5; m2<-7;  m3<-9
pai<-0.7;csi<-0.6
n1<-1000; n2<-500; n3<-1500
ord1<-simcub(n1,m1,pai,csi)
ord2<-simcub(n2,m2,pai,csi)
ord3<-simcub(n3,m3,pai,csi)
listord<-list(ord1,ord2,ord3)
multicub(listord,labels=c("m=5","m=7","m=9"),pos=c(3,1,4))
```

`multicube` *Joint plot of estimated CUBE models in the parameter space*

## Description

Return a plot of estimated CUBE models represented as points in the parameter space, where the overdispersion is labeled.

## Usage

```
multicube(listord,mvett,csiplot=FALSE,paiplot=FALSE,...)
```

## Arguments

| | |
|---|---|
| `listord` | A data matrix, data frame, or list of vectors of ordinal observations (for variables with different number of observations) |
| `mvett` | Vector of number of categories for ordinal variables in `listord` (optional: if missing, the number of categories is retrieved from data: it is advisable to specify it in case some category has zero frequency) |
| `csiplot` | Logical: should $\xi$ or $1-\xi$ be the $y$ coordinate |
| `paiplot` | Logical: should $\pi$ or $1-\pi$ be the $x$ coordinate |
| `...` | Additional arguments to be passed to `plot`, `text`, and `GEM` |

## Value

Fit a CUBE model to list elements, and then by default it returns a plot of the estimated $(1-\pi, 1-\xi)$ as points in the parameter space, labeled with the estimated overdispersion. Depending on `csiplot` and `paiplot` and on desired output, $x$ and $y$ coordinates may be set to $\pi$ and $\xi$, respectively.

## Examples

```
m1<-5; m2<-7;  m3<-9
pai<-0.7;csi<-0.6;phi=0.1
n1<-1000; n2<-500; n3<-1500
ord1<-simcube(n1,m1,pai,csi,phi)
ord2<-simcube(n2,m2,pai,csi,phi)
ord3<-simcube(n3,m3,pai,csi,phi)
listord<-list(ord1,ord2,ord3)
multicube(listord,labels=c("m=5","m=7","m=9"),pos=c(3,1,4),expinform=TRUE)
```

`plotloglikihg` *Plot of the log-likelihood function of the IHG distribution*

## Description

Plot the log-likelihood function of an IHG model fitted to a given absolute frequency distribution, over the whole support of the preference parameter. It returns also the ML estimate.

## Usage

```
plotloglikihg(m,freq)
```

## Arguments

| | |
|---|---|
| `m` | Number of ordinal categories |
| `freq` | Vector of the absolute frequency distribution |

## See Also

`loglikIHG`

## Examples

```
m<-7
freq<-c(828,275,202,178,143,110,101)
max<-plotloglikihg(m,freq)
```

`print.GEM` *S3 method: print for class "GEM"*

## Description

S3 method print for objects of class `GEM`.

## Usage

```
## S3 method for class 'GEM'
print(x, ...)
```

## Arguments

| | |
|---|---|
| `x` | An object of class `GEM` |
| `...` | Other arguments |

## Value

Brief summary results of the fitting procedure, including parameter estimates, their standard errors and the executed call.

## probbit — *Probability distribution of a shifted Binomial random variable*

### Description

Return the shifted Binomial probability distribution.

### Usage

```
probbit(m,csi)
```

### Arguments

| | |
|---|---|
| m | Number of ordinal categories |
| csi | Feeling parameter |

### Value

The vector of the probability distribution of a shifted Binomial model.

### See Also

bitcsi, probcub00

### Examples

```
m<-7
csi<-0.7
pr<-probbit(m,csi)
plot(1:m,pr,type="h",main="Shifted Binomial probability distribution",xlab="Categories")
points(1:m,pr,pch=19)
```

## probcub00 — *Probability distribution of a CUB model without covariates*

### Description

Compute the probability distribution of a CUB model without covariates.

### Usage

```
probcub00(m,pai,csi)
```

### Arguments

| | |
|---|---|
| `m` | Number of ordinal categories |
| `pai` | Uncertainty parameter |
| `csi` | Feeling parameter |

### Value

The vector of the probability distribution of a CUB model.

### References

Piccolo D. (2003). On the moments of a mixture of uniform and shifted binomial random variables. *Quaderni di Statistica*, **5**, 85–104

### See Also

`bitcsi`, `probcub0q`, `probcubp0`, `probcubpq`

### Examples

```
m<-9
pai<-0.3
csi<-0.8
pr<-probcub00(m,pai,csi)
plot(1:m,pr,type="h",main="CUB probability distribution",xlab="Ordinal categories")
points(1:m,pr,pch=19)
```

---

## `probcub0q` *Probability distribution of a CUB model with covariates for the feeling component*

---

### Description

Compute the probability distribution of a CUB model with covariates for the feeling component.

### Usage

```
probcub0q(m,ordinal,W,pai,gama)
```

### Arguments

| | |
|---|---|
| `m` | Number of ordinal categories |
| `ordinal` | Vector of ordinal responses |
| `W` | Matrix of covariates for explaining the feeling component NCOL(Y)+1 to include an intercept term in the model (first entry) |
| `pai` | Uncertainty parameter |
| `gama` | Vector of parameters for the feeling component, whose length equals NCOL(W)+1 to include an intercept term in the model (first entry) |

### Value

A vector of the same length as `ordinal`, whose i-th component is the probability of the i-th observation according to a CUB distribution with the corresponding values of the covariates for the feeling component and coefficients specified in `gama`.

### References

Piccolo D. (2006). Observed Information Matrix for MUB Models, *Quaderni di Statistica*, **8**, 33–78

Piccolo D. and D'Elia A. (2008). A new approach for modelling consumers' preferences, *Food Quality and Preference*, **18**, 247–259

Iannario M. and Piccolo D. (2012). CUB models: Statistical methods and empirical evidence, in: Kenett R. S. and Salini S. (eds.), *Modern Analysis of Customer Surveys: with applications using R*, J. Wiley and Sons, Chichester, 231–258

### See Also

`bitgama`, `probcub00`, `probcubp0`, `probcubpq`

### Examples

```
data(relgoods)
m<-10
naord<-which(is.na(relgoods$Physician))
nacov<-which(is.na(relgoods$Gender))
na<-union(naord,nacov)
ordinal<-relgoods$Physician[-na]
W<-relgoods$Gender[-na]
pai<-0.44; gama<-c(-0.91,-0.7)
pr<-probcub0q(m,ordinal,W,pai,gama)
```

---

## probcube — *Probability distribution of a CUBE model without covariates*

---

### Description

Compute the probability distribution of a CUBE model without covariates.

### Usage

```
probcube(m,pai,csi,phi)
```

### Arguments

| | |
|---|---|
| `m` | Number of ordinal categories |
| `pai` | Uncertainty parameter |
| `csi` | Feeling parameter |
| `phi` | Overdispersion parameter |

## Value

The vector of the probability distribution of a CUBE model without covariates.

## References

Iannario, M. (2014). Modelling Uncertainty and Overdispersion in Ordinal Data, *Communications in Statistics - Theory and Methods*, **43**, 771–786

## See Also

betar, betabinomial

## Examples

```
m<-9
pai<-0.3
csi<-0.8
phi<-0.1
pr<-probcube(m,pai,csi,phi)
plot(1:m,pr,type="h", main="CUBE probability distribution",xlab="Ordinal categories")
points(1:m,pr,pch=19)
```

---

probcubp0 *Probability distribution of a CUB model with covariates for the uncertainty component*

---

## Description

Compute the probability distribution of a CUB model with covariates for the uncertainty component.

## Usage

```
probcubp0(m,ordinal,Y,bet,csi)
```

## Arguments

| | |
|---|---|
| m | Number of ordinal categories |
| ordinal | Vector of ordinal responses |
| Y | Matrix of covariates for explaining the uncertainty component |
| bet | Vector of parameters for the uncertainty component, whose length equals NCOL(Y) + 1 to include an intercept term in the model (first entry) |
| csi | Feeling parameter |

### Value

A vector of the same length as `ordinal`, whose i-th component is the probability of the i-th observation according to a CUB model with the corresponding values of the covariates for the uncertainty component and coefficients for the covariates specified in `bet`.

### References

Piccolo D. (2006). Observed Information Matrix for MUB Models, *Quaderni di Statistica*, **8**, 33–78

Piccolo D. and D'Elia A. (2008). A new approach for modelling consumers' preferences, *Food Quality and Preference*, **18**, 247–259

Iannario M. and Piccolo D. (2012). CUB models: Statistical methods and empirical evidence, in: Kenett R. S. and Salini S. (eds.), *Modern Analysis of Customer Surveys: with applications using R*, J. Wiley and Sons, Chichester, 231–258

### See Also

`bitgama`, `probcub00`, `probcubpq`, `probcub0q`

### Examples

```
data(relgoods)
m<-10
naord<-which(is.na(relgoods$Physician))
nacov<-which(is.na(relgoods$Gender))
na<-union(naord,nacov)
ordinal<-relgoods$Physician[-na]
Y<-relgoods$Gender[-na]
bet<-c(-0.81,0.93); csi<-0.20
probi<-probcubp0(m,ordinal,Y,bet,csi)
```

---

## probcubpq — *Probability distribution of a CUB model with covariates for both feeling and uncertainty*

---

### Description

Compute the probability distribution of a CUB model with covariates for both the feeling and the uncertainty components.

### Usage

```
probcubpq(m,ordinal,Y,W,bet,gama)
```

### Arguments

| | |
|---|---|
| `m` | Number of ordinal categories |
| `ordinal` | Vector of ordinal responses |
| `Y` | Matrix of covariates for explaining the uncertainty component |
| `W` | Matrix of covariates for explaining the feeling component |
| `bet` | Vector of parameters for the uncertainty component, whose length equals NCOL(Y) + 1 to include an intercept term in the model (first entry) |
| `gama` | Vector of parameters for the feeling component, whose length equals NCOL(W)+1 to include an intercept term in the model (first entry) |

### Value

A vector of the same length as `ordinal`, whose i-th component is the probability of the i-th rating according to a CUB distribution with given covariates for both uncertainty and feeling, and specified coefficients vectors `bet` and `gama`, respectively.

### References

Piccolo D. (2006). Observed Information Matrix for MUB Models, *Quaderni di Statistica*, **8**, 33–78
Piccolo D. and D'Elia A. (2008). A new approach for modelling consumers' preferences, *Food Quality and Preference*, **18**, 247–259
Iannario M. and Piccolo D. (2012). CUB models: Statistical methods and empirical evidence, in: Kenett R. S. and Salini S. (eds.), *Modern Analysis of Customer Surveys: with applications using R*, J. Wiley and Sons, Chichester, 231–258

### See Also

`bitgama`, `probcub00`, `probcubp0`, `probcub0q`

### Examples

```
data(relgoods)
m<-10
naord<-which(is.na(relgoods$Physician))
nacov<-which(is.na(relgoods$Gender))
na<-union(naord,nacov)
ordinal<-relgoods$Physician[-na]
W<-Y<-relgoods$Gender[-na]
gama<-c(-0.91,-0.7); bet<-c(-0.81,0.93)
probi<-probcubpq(m,ordinal,Y,W,bet,gama)
```

probcubshe1 *probcubshe1*

## Description

Probability distribution of an extended CUB model with a shelter effect.

## Usage

```
probcubshe1(m,pai1,pai2,csi,shelter)
```

## Arguments

| | |
|---|---|
| m | Number of ordinal categories |
| pai1 | Mixing coefficient for the shifted Binomial component of the mixture distribution |
| pai2 | Mixing coefficient for the discrete Uniform component of the mixture distribution |
| csi | Feeling parameter |
| shelter | Category corresponding to the shelter choice |

## Details

An extended CUB model is a mixture of three components: a shifted Binomial distribution with probability of success $\xi$, a discrete uniform distribution with support $\{1, ..., m\}$, and a degenerate distribution with unit mass at the shelter category (`shelter`).

## Value

The vector of the probability distribution of an extended CUB model with a shelter effect at the shelter category

## References

Iannario M. (2012). Modelling *shelter* choices in a class of mixture models for ordinal responses, *Statistical Methods and Applications*, **21**, 1–22

## See Also

probcubshe2, probcubshe3

## Examples

```
m<-8
pai1<-0.5
pai2<-0.3
csi<-0.4
shelter<-6
pr<-probcubshe1(m,pai1,pai2,csi,shelter)
plot(1:m,pr,type="h",main="Extended CUB probability distribution with shelter effect",
xlab="Ordinal categories")
points(1:m,pr,pch=19)
```

---

# `probcubshe2` *probcubshe2*

---

## Description

Probability distribution of a CUB model with explicit shelter effect

## Usage

```
probcubshe2(m,pai,csi,delta,shelter)
```

## Arguments

| | |
|---|---|
| `m` | Number of ordinal categories |
| `pai` | Uncertainty parameter |
| `csi` | Feeling parameter |
| `delta` | Shelter parameter |
| `shelter` | Category corresponding to the shelter choice |

## Details

A CUB model with explicit shelter effect is a mixture of two components: a CUB distribution with uncertainty parameter $\pi$ and feeling parameter $\xi$, and a degenerate distribution with unit mass at the shelter category (`shelter`) with mixing coefficient specified by $\delta$.

## Value

The vector of the probability distribution of a CUB model with explicit shelter effect.

## References

Iannario M. (2012). Modelling *shelter* choices in a class of mixture models for ordinal responses, *Statistical Methods and Applications*, **21**, 1–22

### See Also

probcubshe1, probcubshe3

### Examples

```
m<-8
pai1<-0.5
pai2<-0.3
csi<-0.4
shelter<-6
delta<-1-pai1-pai2
pai<-pai1/(1-delta)
pr2<-probcubshe2(m,pai,csi,delta,shelter)
plot(1:m,pr2,type="h", main="CUB probability distribution with
explicit shelter effect",xlab="Ordinal categories")
points(1:m,pr2,pch=19)
```

---

## probcubshe3 *probcubshe3*

---

### Description

Probability distribution of a CUB model with explicit shelter effect: satisficing interpretation

### Usage

```
probcubshe3(m,lambda,eta,csi,shelter)
```

### Arguments

| | |
|---|---|
| m | Number of ordinal categories |
| lambda | Mixing coefficient for the shifted Binomial component |
| eta | Mixing coefficient for the mixture of the uncertainty component and the shelter effect |
| csi | Feeling parameter |
| shelter | Category corresponding to the shelter choice |

### Details

The "satisficing interpretation" provides a parametrization for CUB models with explicit shelter effect as a mixture of two components: a shifted Binomial distribution with feeling parameter $\xi$ (meditated choice), and a mixture of a degenerate distribution with unit mass at the shelter category (shelter) and a discrete uniform distribution over $m$ categories, with mixing coefficient specified by $\eta$ (lazy selection of a category).

### Value

The vector of the probability distribution of a CUB model with shelter effect.

## References

Iannario M. (2012). Modelling *shelter* choices in a class of mixture models for ordinal responses, *Statistical Methods and Applications*, **21**, 1–22

## See Also

probcubshe1, probcubshe2

## Examples

```
m<-8
pai1<-0.5
pai2<-0.3
csi<-0.4
shelter<-6
lambda<-pai1
eta<-1-pai2/(1-pai1)
pr3<-probcubshe3(m,lambda,eta,csi,shelter)
plot(1:m,pr3,type="h",main="CUB probability distribution with explicit
shelter effect",xlab="Ordinal categories")
points(1:m,pr3,pch=19)
```

---

`probcush` *Probability distribution of a CUSH model*

---

## Description

Compute the probability distribution of a CUSH model without covariates, that is a mixture of a degenerate random variable with mass at the shelter category and the Uniform distribution.

## Usage

```
probcush(m,delta,shelter)
```

## Arguments

| | |
|---|---|
| `m` | Number of ordinal categories |
| `delta` | Shelter parameter |
| `shelter` | Category corresponding to the shelter choice |

## Value

The vector of the probability distribution of a CUSH model without covariates.

## References

Capecchi S. and Piccolo D. (2017). Dealing with heterogeneity in ordinal responses, *Quality and Quantity*, **51**(5), 2375–2393
Capecchi S. and Iannario M. (2016). Gini heterogeneity index for detecting uncertainty in ordinal data surveys, *Metron*, **74**(2), 223–232

## Examples

```
m<-10
shelter<-1
delta<-0.4
pr<-probcush(m,delta,shelter)
plot(1:m,pr,type="h",xlab="Number of categories")
points(1:m,pr,pch=19)
```

---

# probgecub — *Probability distribution of a GeCUB model*

---

## Description

Compute the probability distribution of a GeCUB model, that is a CUB model with shelter effect with covariates specified for all component.

## Usage

```
probgecub(ordinal,Y,W,X,bet,gama,omega,shelter)
```

## Arguments

| | |
|---|---|
| `ordinal` | Vector of ordinal responses |
| `Y` | Matrix of covariates for explaining the uncertainty component |
| `W` | Matrix of covariates for explaining the feeling component |
| `X` | Matrix of covariates for explaining the shelter effect |
| `bet` | Vector of parameters for the uncertainty component, whose length equals NCOL(Y)+1 to include an intercept term in the model (first entry) |
| `gama` | Vector of parameters for the feeling component, whose length equals NCOL(W)+1 to include an intercept term in the model (first entry) |
| `omega` | Vector of parameters for the shelter effect, whose length equals NCOL(X)+1 to include an intercept term in the model (first entry) |
| `shelter` | Category corresponding to the shelter choice |

## Value

A vector of the same length as `ordinal`, whose i-th component is the probability of the i-th observation according to a GeCUB model with the corresponding values of the covariates for all the components and coefficients specified in `bet`, `gama`, `omega`.

## References

Iannario M. and Piccolo D. (2016b). A generalized framework for modelling ordinal data. *Statistical Methods and Applications*, **25**, 163–189.

---

`probihg` *Probability distribution of an IHG model*

---

## Description

Compute the probability distribution of an IHG model (Inverse Hypergeometric) without covariates.

## Usage

```
probihg(m,theta)
```

## Arguments

| | |
|---|---|
| `m` | Number of ordinal categories |
| `theta` | Preference parameter |

## Value

The vector of the probability distribution of an IHG model.

## References

D'Elia A. (2003). Modelling ranks using the inverse hypergeometric distribution, *Statistical Modelling: an International Journal*, **3**, 65–78

## Examples

```
m<-10
theta<-0.30
pr<-probihg(m,theta)
plot(1:m,pr,type="h",xlab="Ordinal categories")
points(1:m,pr,pch=19)
```

`probihgcovn` *Probability distribution of an IHG model with covariates*

## Description

Given a vector of $n$ ratings over $m$ categories, it returns a vector of length $n$ whose i-th element is the probability of observing the i-th rating for the corresponding IHG model with parameter $\theta_i$, obtained via logistic link with covariates and coefficients.

## Usage

```
probihgcovn(m,ordinal,U,nu)
```

## Arguments

| | |
|---|---|
| `m` | Number of ordinal categories |
| `ordinal` | Vector of ordinal responses |
| `U` | Matrix of selected covariates for explaining the preference parameter |
| `nu` | Vector of coefficients for covariates, whose length equals NCOL(U)+1 to include an intercept term in the model (first entry) |

## Details

The matrix $U$ is expanded with a vector with entries equal to 1 in the first column to include an intercept term in the model.

## See Also

`probihg`

## Examples

```
n<-100
m<-7
theta<-0.30
ordinal<-simihg(n,m,theta)
U<-sample(c(0,1),n,replace=TRUE)
nu<-c(0.12,-0.5)
pr<-probihgcovn(m,ordinal,U,nu)
```

relgoods *Relational goods and Leisure time dataset*

## Description

Dataset consists of the results of a survey aimed at measuring the evaluation of people living in the metropolitan area of Naples, Italy, with respect to of relational goods and leisure time collected in December 2014. Every participant was asked to assess on a 10 point ordinal scale his/her personal score for several relational goods (for instance, time dedicated to friends and family) and to leisure time. In addition, the survey asked respondents to self-evaluate their level of happiness by marking a sign along a horizontal line of 110 millimeters according to their feeling, with the left-most extremity standing for "extremely unhappy", and the right-most extremity corresponding to the status "extremely happy".

## Usage

```
data(relgoods)
```

## Format

The description of subjects' covariates is the following:

`ID` An identification number

`Gender` A factor with levels: 0 = man, 1 = woman

`BirthMonth` A variable indicating the month of birth of the respondent

`BirthYear` A variable indicating the year of birth of the respondent

`Family` A factor variable indicating the number of members of the family

`Year.12` A factor with levels: 1 = if there is any child aged less than 12 in the family, 0 = otherwise

`EducationDegree` A factor with levels: 1 = compulsory school, 2 = high school diploma, 3 = Graduated-Bachelor degree, 4 = Graduated-Master degree, 5 = Post graduated

`MaritalStatus` A factor with levels: 1 = Unmarried, 2 = Married/Cohabitee, 3 = Separated/Divorced, 4 = Widower

`Residence` A factor with levels: 1 = City of Naples, 2 = District of Naples, 3 = Others Campania, 4 = Others Italia, 5 = Foreign countries

`Glasses` A factor with levels: 1 = wearing glasses or contact lenses, 0 = otherwise

`RightHand` A factor with levels: 1 = right-handed, 0 = left-handed

`Smoking` A factor with levels: 1 = smoker, 0 = not smoker

`WalkAlone` A factor with levels: 1 = usually walking alone, 0 = usually walking in company

`job` A factor with levels: 1 = Not working, 2 = Retired, 3 = occasionally, 4 = fixed-term job, 5 = permanent job

`PlaySport` A factor with levels: 1 = Not playing any sport, 2 = Yes, individual sport, 3 = Yes, team sport

`Pets` A factor with levels: 1 = owning a pet, 0 = not owning any pet

1. Respondents were asked to evaluate the following items on a 10 point Likert scale, ranging from 1 = "never, at all" to 10 = "always, a lot":

   `WalkOut` How often the respondent goes out for a walk

   `Parents` How often respondent talks at least to one of his/her parents

   `MeetRelatives` How often respondent meets his/her relatives

   `Association` Frequency of involvement in volunteering or different kinds of associations/parties, etc

   `RelFriends` Quality of respondent's relationships with friends

   `RelNeighbours` Quality of the relationships with neighbors

   `NeedHelp` Easiness in asking help whenever in need

   `Environment` Level of comfort with the surrounding environment

   `Safety` Level of safety in the streets

   `EndofMonth` Family making ends meet

   `MeetFriend` Number of times the respondent met his/her friends during the month preceding the interview

   `Physician` Importance of the kindness/simpathy in the selection of respondent's physician

   `Happiness` Each respondent was asked to mark a sign on a 110mm horizontal line according to his/her feeling of happiness (left endpoint corresponding to completely unhappy, rightmost endpoint corresponding to extremely happy

2. The same respondents were asked to score the activities for leisure time listed below, according to their involvement/degree of amusement, on a 10 point Likert scale ranging from 1 = "At all, nothing, never" to 10 = "Totally, extremely important, always":

   `Videogames`

   `Reading`

   `Cinema`

   `Drawing`

   `Shopping`

   `Writing`

   `Bicycle`

   `Tv`

   `StayWFriend` Spending time with friends

   `Groups` Taking part to associations, meetings, etc.

   `Walking`

   `HandWork` Hobby, gardening, sewing, etc.

   `Internet`

   `Sport`

   `SocialNetwork`

   `Gym`

   `Quiz` Crosswords, sudoku, etc.

   `MusicInstr` Playing a musical instrument

   `GoAroundCar` Hanging out by car

   `Dog` Walking out the dog

   `GoOutEat` Go to restaurants/pubs

## Details

Period of data collection: December 2014
Mode of collection: questionnaire
Number of observations: 2459
Number of subjects' covariates: 16
Number of analyzed items: 34
Warning: with a limited number of missing values

---

simcub *Simulation routine for CUB models*

---

## Description

Generate $n$ pseudo-random observations following the given CUB distribution.

## Usage

```
simcub(n,m,pai,csi)
```

## Arguments

| | |
|---|---|
| n | Number of simulated observations |
| m | Number of ordinal categories |
| pai | Uncertainty parameter |
| csi | Feeling parameter |

## See Also

probcub00

## Examples

```
n<-300
m<-9
pai<-0.4
csi<-0.7
simulation<-simcub(n,m,pai,csi)
plot(table(simulation),xlab="Ordinal categories",ylab="Frequencies")
```

simcube *Simulation routine for CUBE models*

### Description

Generate $n$ pseudo-random observations following the given CUBE distribution.

### Usage

```
simcube(n,m,pai,csi,phi)
```

### Arguments

| | |
|---|---|
| n | Number of simulated observations |
| m | Number of ordinal categories |
| pai | Uncertainty parameter |
| csi | Feeling parameter |
| phi | Overdispersion parameter |

### See Also

probcube

### Examples

```
n<-300
m<-9
pai<-0.7
csi<-0.4
phi<-0.1
simulation<-simcube(n,m,pai,csi,phi)
plot(table(simulation),xlab="Ordinal categories",ylab="Frequencies")
```

simcubshe *Simulation routine for CUB models with shelter effect*

### Description

Generate $n$ pseudo-random observations following the given CUB distribution with shelter effect.

### Usage

```
simcubshe(n,m,pai,csi,delta,shelter)
```

### Arguments

| | |
|---|---|
| n | Number of simulated observations |
| m | Number of ordinal categories |
| pai | Uncertainty parameter |
| csi | Feeling parameter |
| delta | Shelter parameter |
| shelter | Category corresponding to the shelter choice |

### See Also

probcubshe1, probcubshe2, probcubshe3

### Examples

```
n<-300
m<-9
pai<-0.7
csi<-0.3
delta<-0.2
shelter<-3
simulation<-simcubshe(n,m,pai,csi,delta,shelter)
plot(table(simulation),xlab="Ordinal categories",ylab="Frequencies")
```

---

## simcush *Simulation routine for CUSH models*

---

### Description

Generate $n$ pseudo-random observations following the distribution of a CUSH model without covariates.

### Usage

```
simcush(n,m,delta,shelter)
```

### Arguments

| | |
|---|---|
| n | Number of simulated observations |
| m | Number of ordinal categories |
| delta | Shelter parameter |
| shelter | Category corresponding to the shelter choice |

### See Also

probcush

## Examples

```
n<-200
m<-7
delta<-0.3
shelter<-3
simulation<-simcush(n,m,delta,shelter)
plot(table(simulation),xlab="Ordinal categories",ylab="Frequencies")
```

---

simihg *Simulation routine for IHG models*

---

## Description

Generate $n$ pseudo-random observations following the given IHG distribution.

## Usage

```
simihg(n,m,theta)
```

## Arguments

| | |
|---|---|
| n | Number of simulated observations |
| m | Number of ordinal categories |
| theta | Preference parameter |

## See Also

probihg

## Examples

```
n<-300
m<-9
theta<-0.4
simulation<-simihg(n,m,theta)
plot(table(simulation),xlab="Number of categories",ylab="Frequencies")
```

## summary.GEM *S3 method: summary for class "GEM"*

### Description

S3 method summary for objects of class GEM.

### Usage

```
## S3 method for class 'GEM'
summary(object, correlation = FALSE, ...)
```

### Arguments

| | |
|---|---|
| object | An object of class GEM |
| correlation | Logical: should the estimated correlation matrix be returned? Default is FALSE |
| ... | Other arguments |

### Value

Extended summary results of the fitting procedure, including parameter estimates, their standard errors and Wald statistics, maximized log-likelihood compared with that of the saturated model and of a Uniform sample. AIC, BIC and ICOMP indeces are also displayed for model selection. Execution time and number of exectued iterations for the fitting procedure are aslo returned.

### Examples

```
model<-GEM(Formula(MeetRelatives~0|0|0),family="cube",data=relgoods)
summary(model,correlation=TRUE,digits=4)
```

## univer *Evaluation of the Orientation Services 2002*

### Description

A sample survey on students evaluation of the Orientation services was conducted across the 13 Faculties of University of Naples Federico II in five waves: participants were asked to express their ratings on a 7 point scale (1 = "very unsatisfied", 7 = "extremely satisfied"). Here dataset collected during 2002 is loaded.

### Usage

```
data(univer)
```

## Format

The description of subjects' covariates is:

`Faculty` A factor variable, with levels ranging from 1 to 13 indicating the coding for the different university faculties

`Freqserv` A factor with levels: 0 = for not regular users, 1 = for regular users

`Age` Variable indicating the age of the respondent in years

`Gender` A factor with levels: 0 = man, 1 = woman

`Diploma` A factor with levels: 1 = classic studies, 2 = scientific studies, 3 = linguistic, 4 = Professional, 5 = Technical/Accountancy, 6 = others

`Residence` A factor with levels: 1 = city NA, 2 = district NA, 3 = others

`ChangeFa` A factor with levels: 1 = changed faculty, 2 = not changed faculty

Analyzed ordinal variables (Likert ordinal scale): 1 = "extremely unsatisfied", 2 = "very unsatisfied", 3 = "unsatisfied", 4 = "indifferent", 5 = "satisfied", 6 = "very satisfied", 7 = "extremely satisfied"

`Informat` Level of satisfaction about the collected information

`Willingn` Level of satisfaction about the willingness of the staff

`Officeho` Judgment about the Office hours

`Competen` Judgement about the competence of the staff

`Global` Global satisfaction

## Details

Period of data collection: 2002
Mode of collection: questionnaire
Number of observations: 2179
Number of subjects' covariates: 7
Number of analyzed items: 5

---

`varcub00` *Variance of CUB models without covariates*

---

## Description

Compute the variance of a CUB model without covariates.

## Usage

```
varcub00(m,pai,csi)
```

### Arguments

| | |
|---|---|
| m | Number of ordinal categories |
| pai | Uncertainty parameter |
| csi | Feeling parameter |

### References

Piccolo D. (2003). On the moments of a mixture of uniform and shifted binomial random variables. *Quaderni di Statistica*, **5**, 85–104

### See Also

expcub00, probcub00

### Examples

```
m<-9
pai<-0.6
csi<-0.5
varcub<-varcub00(m,pai,csi)
```

---

## varcube *Variance of CUBE models without covariates*

---

### Description

Compute the variance of a CUBE model without covariates.

### Usage

```
varcube(m,pai,csi,phi)
```

### Arguments

| | |
|---|---|
| m | Number of ordinal categories |
| pai | Uncertainty parameter |
| csi | Feeling parameter |
| phi | Overdispersion parameter |

### References

Iannario, M. (2014). Modelling Uncertainty and Overdispersion in Ordinal Data, *Communications in Statistics - Theory and Methods*, **43**, 771–786

### See Also

probcube, expcube

## Examples

```
m<-7
pai<-0.8
csi<-0.2
phi<-0.05
varianceCUBE<-varcube(m,pai,csi,phi)
```

---

`varmatCUB` *Variance-covariance matrix for CUB models*

---

## Description

Compute the variance-covariance matrix of parameter estimates for CUB models with or without covariates for the feeling and the uncertainty parameter, and for extended CUB models with shelter effect.

## Usage

```
varmatCUB(ordinal,m,param,Y=0,W=0,X=0,shelter=0)
```

## Arguments

| | |
|---|---|
| `ordinal` | Vector of ordinal responses |
| `m` | Number of ordinal categories |
| `param` | Vector of parameters for the specified CUB model |
| `Y` | Matrix of selected covariates to explain the uncertainty component (default: no covariate is included in the model) |
| `W` | Matrix of selected covariates to explain the feeling component (default: no covariate is included in the model) |
| `X` | Matrix of selected covariates to explain the shelter effect (default: no covariate is included in the model) |
| `shelter` | Category corresponding to the shelter choice (default: no shelter effect is included in the model) |

## Details

The function checks if the variance-covariance matrix is positive-definite: if not, it returns a warning message and produces a matrix with NA entries. No missing value should be present neither for `ordinal` nor for covariate matrices: thus, deletion or imputation procedures should be preliminarily run.

## References

Piccolo D. (2006). Observed Information Matrix for MUB Models, *Quaderni di Statistica*, **8**, 33–78
Iannario, M. (2012). Modelling shelter choices in ordinal data surveys. *Statistical Modelling and Applications*, **21**, 1–22
Iannario M. and Piccolo D. (2016b). A generalized framework for modelling ordinal data. *Statistical Methods and Applications*, **25**, 163–189.

## See Also

vcov, cormat

## Examples

```
data(univer)
m<-7
### CUB model with no covariate
pai<-0.87; csi<-0.17
param<-c(pai,csi)
varmat<-varmatCUB(univer$global,m,param)
#######################
### and with covariates for feeling
data(univer)
m<-7
pai<-0.86; gama<-c(-1.94,-0.17)
param<-c(pai,gama)
ordinal<-univer$willingn; W<-univer$gender
varmat<-varmatCUB(ordinal,m,param,W)
#######################
### CUB model with uncertainty covariates
data(relgoods)
m<-10
naord<-which(is.na(relgoods$Physician))
nacov<-which(is.na(relgoods$Gender))
na<-union(naord,nacov)
ordinal<-relgoods$Physician[-na]
Y<-relgoods$Gender[-na]
bet<-c(-0.81,0.93); csi<-0.20
varmat<-varmatCUB(ordinal,m,param=c(bet,csi),Y=Y)
#######################
### and with covariates for both parameters
data(relgoods)
m<-10
naord<-which(is.na(relgoods$Physician))
nacov<-which(is.na(relgoods$Gender))
na<-union(naord,nacov)
ordinal<-relgoods$Physician[-na]
W<-Y<-relgoods$Gender[-na]
gama<-c(-0.91,-0.7); bet<-c(-0.81,0.93)
varmat<-varmatCUB(ordinal,m,param=c(bet,gama),Y=Y,W=W)
#######################
```

```
### Variance-covariance for a CUB model with shelter
m<-8; n<-300
pai1<-0.5; pai2<-0.3; csi<-0.4
shelter<-6
pr<-probcubshe1(m,pai1,pai2,csi,shelter)
ordinal<-sample(1:m,n,prob=pr,replace=TRUE)
param<-c(pai1,pai2,csi)
varmat<-varmatCUB(ordinal,m,param,shelter=shelter)
```

---

varmatCUBE *Variance-covariance matrix for CUBE models*

---

## Description

Compute the variance-covariance matrix of parameter estimates for CUBE models when no covariate is specified, or when covariates are included for all the three parameters.

## Usage

```
varmatCUBE(ordinal,m,param,Y=0,W=0,Z=0,expinform=FALSE)
```

## Arguments

| | |
|---|---|
| `ordinal` | Vector of ordinal responses |
| `m` | Number of ordinal categories |
| `param` | Vector of parameters for the specified CUBE model |
| `Y` | Matrix of selected covariates to explain the uncertainty component (default: no covariate is included in the model) |
| `W` | Matrix of selected covariates to explain the feeling component (default: no covariate is included in the model) |
| `Z` | Matrix of selected covariates to explain the overdispersion component (default: no covariate is included in the model) |
| `expinform` | Logical: if TRUE and no covariate is included in the model, the function returns the expected variance-covariance matrix (default is FALSE: the function returns the observed variance-covariance matrix) |

## Details

The function checks if the variance-covariance matrix is positive-definite: if not, it returns a warning message and produces a matrix with NA entries. No missing value should be present neither for `ordinal` nor for covariate matrices: thus, deletion or imputation procedures should be preliminarily run.

### References

Iannario, M. (2014). Modelling Uncertainty and Overdispersion in Ordinal Data, *Communications in Statistics - Theory and Methods*, **43**, 771–786
Piccolo D. (2015). Inferential issues for CUBE models with covariates, *Communications in Statistics. Theory and Methods*, **44**(23), 771–786.

### See Also

vcov, cormat

### Examples

```
m<-7; n<-500
pai<-0.83; csi<-0.19; phi<-0.045
ordinal<-simcube(n,m,pai,csi,phi)
param<-c(pai,csi,phi)
varmat<-varmatCUBE(ordinal,m,param)
##########################
### Including covariates
data(relgoods)
m<-10
naord<-which(is.na(relgoods$Tv))
nacov<-which(is.na(relgoods$BirthYear))
na<-union(naord,nacov)
age<-2014-relgoods$BirthYear[-na]
lage<-log(age)-mean(log(age))
Y<-W<-Z<-lage
ordinal<-relgoods$Tv[-na]
estbet<-c(0.18,1.03); estgama<-c(-0.6,-0.3); estalpha<-c(-2.3,0.92)
param<-c(estbet,estgama,estalpha)
varmat<-varmatCUBE(ordinal,m,param,Y=Y,W=W,Z=Z,expinform=TRUE)
```

---

## vcov.GEM — *S3 method vcov() for class "GEM"*

### Description

S3 method: vcov for objects of class GEM.

### Usage

```
## S3 method for class 'GEM'
vcov(object, ...)
```

### Arguments

| | |
|---|---|
| object | An object of class GEM |
| ... | Other arguments |

## Value

Variance-covariance matrix of the final ML estimates for parameters of the fitted GEM model. It returns the square of the estimated standard error for CUSH and IHG models with no covariates.

## See Also

`varmatCUB`, `varmatCUBE`, `GEM`

# Index