# Package 'mult.latent.reg'

March 21, 2024

Type Package

Title Regression and Clustering in Multivariate Response Scenarios

Version 0.1.7

#### **Description**

Fitting multivariate response models with random effects on one or two levels; whereby the (one-dimensional) random effect represents a latent variable approximating the multivariate space of outcomes, after possible adjustment for covariates. The method is particularly useful for multivariate, highly correlated outcome variables with unobserved heterogeneities. Applications include regression with multivariate responses, as well as multivariate clustering or ranking problems. See Zhang and Einbeck (2024) <doi:10.1007/s42519-023-00357-0>.

License GPL-3

Imports mytnorm, stats, matrixStats, utils, lme4

**Encoding UTF-8** 

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**Depends** R (>= 3.5.0)

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## **Description**

The data were recorded via 4D ultrasound scans from 40 fetuses (20 before Covid and 20 during Covid) at 32 weeks gestation, and consist of the number of movements each fetus carries out in relation to the recordable scan length.

#### Usage

```
data(fetal_covid_data)
```

#### **Format**

An object of class "data.frame"

**UpperFaceMovements** Inner Brow Raiser, Outer Brow Raiser, Brow Lower, Cheek Raiser, Nose Wrinkle.

Headmovements Turn Right, Turn Left, Up, Down.

**MouthMovements** Upper Lip Raiser, Nasolabial Furrow, Lip Puller, Lower Lip Depressor, Lip Pucker, Tongue Show, Lip Stretch, Lip Presser, Lip Suck, Lips Parting, Jaw Drop, Mouth Stretch.

TouchMovements Upper Face, Side Face, Lower Face, Mouth Area.

**EyeBlink** All scans were coded for eye blink.

status\_bi "during the pandemic" is coded by 1, "before the pandemic" is coded by 0.

status specifies whether it is during or before the pandemic.

#### References

Reissland, N., Ustun, B. and Einbeck, J. (2024). The effects of lockdown during the COVID-19 pandemic on fetal movement profiles. BMC Pregnancy and Childbirth, 24(1), 1-7.

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#### **Examples**

```
data(fetal_covid_data)
head(fetal_covid_data)
```

IALS\_data

International Adult Literacy Survey (IALS) for 13 countries

## **Description**

The data is obtained from the International Adult Literacy Survey (IALS), collected in 13 countries on Prose, Document, and Quantitative scales between 1994 and 1995. The data are reported as the percentage of individuals who could not reach a basic level of literacy in each country.

## Usage

```
data(IALS_data)
```

#### **Format**

An object of class "data.frame"

**Prose** On prose scale, the percentage of individuals who could not reach a basic level of literacy in each country.

**Document** On document scale, the percentage of individuals who could not reach a basic level of literacy in each country.

**Quantitative** On quantitative scale, the percentage of individuals who could not reach a basic level of literacy in each country.

Country Specify the country

Gender Specify the gender

#### References

Sofroniou, N., Hoad, D., & Einbeck, J. (2008). League tables for literacy survey data based on random effect models. In: Proceedings of the 23rd International Workshop on Statistical Modelling, Utrecht; pp. 402-405.

```
data(IALS_data)
head(IALS_data)
```

4 mult.em\_1level

mult.em\_1level

EM algorithm for multivariate one level model with covariates

## **Description**

This function is used to obtain the Maximum Likelihood Estimates (MLE) using the EM algorithm for one-level multivariate data. The estimates enable users to conduct clustering, ranking, and simultaneous dimension reduction on the multivariate dataset. Furthermore, when covariates are included, the function supports the fitting of multivariate response models, expanding its utility for regression analysis. The details of the model used in this function can be found in Zhang and Einbeck (2024).

## **Arguments**

data	A data set object; we denote the dimension to be $m$ .
V	Covariate(s).
K	Number of mixture components, the default is $K=2$ . Note that when $K=1$ , $z$ and beta will be 0.
steps	Number of iterations, the default is steps = 20.
start	Containing parameters involved in the proposed model (p, alpha, z, beta, sigma, gamma) in a list, the starting values can be obtained through the use of start_em.  More details can be found in start_em.
option	Four options for selecting the starting values for the parameters in the model. The default is option = 1. More details can be found in start_em.
var_fun	There are four types of variance specifications; var_fun = 1, the same diagonal variance specification to all K components of the mixture; var_fun = 2, different diagonal variance matrices for different components. var_fun = 3, the same full (unrestricted) variance for all components. var_fun = 4, different full (unrestricted) variance matrices for different components. The default is var_fun = 2.

#### Value

The estimated parameters in the model  $x_i = \alpha + \beta z_k + \Gamma v_i + \varepsilon_i$  obtained through the EM algorithm at the convergence.

р	The estimates for the parameter $\pi_k$ , which is a vector of length $K$ .
alpha	The estimates for the parameter $\alpha$ , which is a vector of length $m$ .
z	The estimates for the parameter $z_k$ , which is a vector of length $K$ .
beta	The estimates for the parameter $\beta$ , which is a vector of length $m$ .
gamma	The estimates for the parameter $\Gamma$ , which is a matrix.

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sigma The estimates for the parameter  $\Sigma_k$ . When var\_fun = 1,  $\Sigma_k$  is a diagonal matrix and  $\Sigma_k = \Sigma$ , and we obtain a vector of the diagonal elements; When var\_fun

= 2,  $\Sigma_k$  is a diagonal matrix, and we obtain K vectors of the diagonal elements; When var\_fun = 3,  $\Sigma_k$  is a full variance-covariance matrix,  $\Sigma_k = \Sigma$ , and we obtain a matrix  $\Sigma$ ; When var\_fun = 4,  $\Sigma_k$  is a full variance-covariance matrix,

and we obtain K different matrices  $\Sigma_k$ .

W The posterior probability matrix.

loglikelihood The approximated log-likelihood of the fitted model.

disparity The disparity (-2logL) of the fitted model.

number\_parameters

The number of parameters estimated in the EM algorithm.

AIC The AIC value (-2logL + 2number\_parameters).

BIC The BIC value (-2logL + number\_parameters\*log(n)), where n is the number

of observations.

starting\_values

A list of starting values for parameters used in the EM algorithm.

#### References

Zhang, Y. and Einbeck, J. (2024). A Versatile Model for Clustered and Highly Correlated Multivariate Data. J Stat Theory Pract 18(5).doi:10.1007/s42519023003570

#### See Also

```
mult.reg_1level.
```

```
##example for data without covariates.
data(faithful)
res <- mult.em_1level(faithful,K=2,steps = 10,var_fun = 1)
## Graph showing the estimated one-dimensional space with cluster centers in red and alpha in green.
x <- res$alpha[1]+res$beta[1]*res$z
y <- res$alpha[2]+res$beta[2]*res$z
plot(faithful,col = 8)
points(x=x[1],y=y[1],type = "p",col = "red",pch = 17)
points(x=x[2],y=y[2],type = "p",col = "red",pch = 17)
points(x=res$alpha[1],y=res$alpha[2],type = "p",col = "darkgreen",pch = 4)
slope <- (y[2]-y[1])/(x[2]-x[1])
intercept <- y[1]-slope*x[1]</pre>
abline(intercept, slope, col="red")
##Graph showing the original data points being assigned to different
 ##clusters according to the Maximum a posterior (MAP) rule.
index <- apply(res$W, 1, which.max)</pre>
faithful_grouped <- cbind(faithful,index)</pre>
colors <- c("#FDAE61", "#66BD63")</pre>
```

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mult.em\_2level

EM algorithm for multivariate two level model with covariates

## **Description**

This function extends the one-level version mult.em\_1level, and it is designed to obtain Maximum Likelihood Estimates (MLE) using the EM algorithm for nested (structured) multivariate data, e.g. multivariate test scores (such as on numeracy, literacy) of students nested in different classes or schools. The resulting estimates can be applied for clustering or constructing league tables (ranking of observations). With the inclusion of covariates, the model allows fitting a multivariate response model for further regression analysis. Detailed information about the model used in this function can be found in Zhang et al. (2023).

#### **Arguments**

data	A data set object; we denote the dimension to be $m$ .
V	Covariate(s).
K	Number of mixture components, the default is $K = 2$ . Note that when $K = 1$ , $z$ and beta will be 0.
steps	Number of iterations, the default is steps = 20.
start	Containing parameters involved in the proposed model (p, alpha, z, beta, sigma, gamma) in a list, the starting values can be obtained through the use of start_em. More details can be found in start_em.
option	Four options for selecting the starting values for the parameters in the model. The default is option = 1. More details can be found in start_em.
var_fun	There are two types of variance specifications; var_fun = 1, the same diagonal variance specification to all K components of the mixture; var_fun = 2, different diagonal variance matrices for different components; The default is var_fun = 2.

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#### Value

The estimated parameters in the model  $x_{ij} = \alpha + \beta z_k + \Gamma v_{ij} + \varepsilon_{ij}$  obtained through the EM algorithm, where the upper-level unit is indexed by i, and the lower-level unit is indexed by j.

The estimates for the parameter  $\pi_k$ , which is a vector of length K. alpha The estimates for the parameter  $\alpha$ , which is a vector of length m. The estimates for the parameter  $z_k$ , which is a vector of length K. The estimates for the parameter  $\beta$ , which is a vector of length m. beta The estimates for the parameter  $\Gamma$ , which is a matrix. gamma sigma The estimates for the parameter  $\Sigma_k$ . When var\_fun = 1,  $\Sigma_k$  is a diagonal matrix and  $\Sigma_k = \Sigma$ , and we obtain a vector of the diagonal elements; When var\_fun = 2,  $\Sigma_k$  is a diagonal matrix, and we obtain K vectors of the diagonal elements. The posterior probability matrix. The approximated log-likelihood of the fitted model. loglikelihood disparity The disparity (-2logL) of the fitted model. number\_parameters The number of parameters estimated in the EM algorithm. AIC

The AIC value (-2logL + 2number\_parameters).

starting\_values

A list of starting values for parameters used in the EM algorithm.

#### References

Zhang, Y., Einbeck, J. and Drikvandi, R. (2023). A multilevel multivariate response model for data with latent structures. In: Proceedings of the 37th International Workshop on Statistical Modelling, pages 343-348. Link on RG: https://www.researchgate.net/publication/375641972\_A\_ multilevel\_multivariate\_response\_model\_for\_data\_with\_latent\_structures

#### See Also

```
mult.reg_2level.
```

```
##examples for data without covariates.
data(trading_data)
set.seed(49)
trade_res <- mult.em_2level(trading_data, K=4, steps = 10, var_fun = 2)</pre>
i_1 <- apply(trade_res$W, 1, which.max)</pre>
colors <- c("#FF6600","#66BD63", "lightpink","purple")</pre>
plot(trading_data[,-3],pch = 1, col = colors[factor(ind_certain)])
legend("topleft", legend=c("Mass point 1", "Mass point 2", "Mass point 3", "Mass point 4"),
col=c("#FF6600","purple","#66BD63","lightpink"),pch = 1, cex=0.8)
```

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```
###The Twins data
library(lme4)
set.seed(26)
twins_res <- mult.em_2level(twins_data[,c(1,2,3)],v=twins_data[,c(4,5,6)],
K=2, steps = 20, var_fun = 2)
coeffs <- twins_res$gamma
##Compare to the estimated coefficients obtained using individual two-level models (lmer()).
summary(lmer(SelfTouchCodable ~ Depression + PSS + Anxiety + (1 | id) ,
data=twins_data, REML = TRUE))$coefficients[2,1]</pre>
```

mult.latent.reg

Regression and Clustering in Multivariate Response Scenarios

## Description

This package implements methodology for the estimation of multivariate response models with random effects on one or two levels; whereby the (one-dimensional) random effect represents a latent variable approximating the multivariate space of outcomes, after possible adjustment for covariates. The estimation methodology makes use of a nonparametric maximum likelihood-type approach, where the random effect distribution is approximated by a discrete mixture, hence allowing the use of the EM algorithm for the estimation of all model parameters. The method is particularly useful for multivariate, highly correlated outcome variables with unobserved heterogeneities. Applications include regression with multivariate responses, as well as multivariate clustering or ranking problems. The details of the models can be found in Zhang and Einbeck (2024) and Zhang et al. (2023). The main functions are mult.em\_1level and mult.em\_2level for the fitting of the raw models, as well as envelope functions mult.reg\_1level and mult.reg\_2level which facilitate iterative runs of the algorithm with a view to finding optimal starting points, with help by function start\_em.

#### **Details**

Package: mult.latent.reg

Type: Package License: GPL-3

#### Author(s)

Yingjuan Zhang <yingjuan.zhang@durham.ac.uk>
Jochen Einbeck

#### References

Zhang, Y., Einbeck, J., and Drikvandi, R. (2023). A multilevel multivariate response model for data with latent structures. In: Proceedings of the 37th International Workshop on Statistical Modelling, Dortmund; pages 343-348. Link on RG: https://www.researchgate.net/publication/375641972\_A\_multilevel\_multivariate\_response\_model\_for\_data\_with\_latent\_structures.

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Zhang, Y. and Einbeck, J. (2024). A Versatile Model for Clustered and Highly Correlated Multivariate Data. J Stat Theory Pract 18(5).doi:10.1007/s42519023003570

mult.reg\_1level Selecting the best results for multivariate one level model

## Description

This wrapper function runs multiple times the function mult.em\_llevel for fitting Zhang and Einbeck's (2024) multivariate response models with one-level random effect, and select the best results with the smallest AIC value.

## Arguments

A data set object; we denote the dimension of a data set to be m.  Covariate(s).  K Number of mixture components, the default is K = 2.  steps Number of iterations within each num_runs, the default is steps = 20.  num_runs Number of function iteration runs, the default is num_runs = 10.  start Containing parameters involved in the proposed model (p, alpha, z, beta, sigma gamma) in a list, the starting values can be obtained through the use of start_em.  More details can be found in start_em.  option Four options for selecting the starting values for the parameters in the model. The default is option = 1. More details can be found in start_em.  var_fun There are four types of variance specifications; var_fun = 1, the same diagonal variance specification to all K components of the mixture; var_fun = 2, different diagonal variance matrices for different components. var_fun = 3, the same full (unrestricted) variance for all components. var_fun = 4, different full (unrestricted) variance matrices for different components. The default is var_fun = 2.		
Number of mixture components, the default is K = 2.  Steps Number of iterations within each num_runs, the default is steps = 20.  Num_runs Number of function iteration runs, the default is num_runs = 10.  Start Containing parameters involved in the proposed model (p, alpha, z, beta, sigma gamma) in a list, the starting values can be obtained through the use of start_em. More details can be found in start_em.  Option Four options for selecting the starting values for the parameters in the model. The default is option = 1. More details can be found in start_em.  var_fun There are four types of variance specifications; var_fun = 1, the same diagonal variance specification to all K components of the mixture; var_fun = 2, different diagonal variance matrices for different components. var_fun = 3, the same full (unrestricted) variance for all components. var_fun = 4, different full (unrestricted) variance matrices for different components. The default is var_fun	data	A data set object; we denote the dimension of a data set to be $m$ .
Number of iterations within each num_runs, the default is steps = 20.  Number of function iteration runs, the default is num_runs = 10.  start Containing parameters involved in the proposed model (p, alpha, z, beta, sigma gamma) in a list, the starting values can be obtained through the use of start_em.  More details can be found in start_em.  option Four options for selecting the starting values for the parameters in the model. The default is option = 1. More details can be found in start_em.  var_fun There are four types of variance specifications; var_fun = 1, the same diagonal variance specification to all K components of the mixture; var_fun = 2, different diagonal variance matrices for different components. var_fun = 3, the same full (unrestricted) variance for all components. var_fun = 4, different full (unrestricted) variance matrices for different components. The default is var_fun	V	Covariate(s).
num_runs  Number of function iteration runs, the default is num_runs = 10.  start  Containing parameters involved in the proposed model (p, alpha, z, beta, sigma gamma) in a list, the starting values can be obtained through the use of start_em.  More details can be found in start_em.  option  Four options for selecting the starting values for the parameters in the model. The default is option = 1. More details can be found in start_em.  var_fun  There are four types of variance specifications; var_fun = 1, the same diagonal variance specification to all K components of the mixture; var_fun = 2, different diagonal variance matrices for different components. var_fun = 3, the same full (unrestricted) variance for all components. var_fun = 4, different full (unrestricted) variance matrices for different components. The default is var_fun	K	Number of mixture components, the default is K = 2.
Containing parameters involved in the proposed model (p, alpha, z, beta, sigma gamma) in a list, the starting values can be obtained through the use of start_em.  More details can be found in start_em.  Four options for selecting the starting values for the parameters in the model. The default is option = 1. More details can be found in start_em.  Var_fun  There are four types of variance specifications; var_fun = 1, the same diagonal variance specification to all K components of the mixture; var_fun = 2, different diagonal variance matrices for different components. var_fun = 3, the same full (unrestricted) variance for all components. var_fun = 4, different full (unrestricted) variance matrices for different components. The default is var_fun	steps	Number of iterations within each num_runs, the default is steps = 20.
gamma) in a list, the starting values can be obtained through the use of start_em.  More details can be found in start_em.  Four options for selecting the starting values for the parameters in the model. The default is option = 1. More details can be found in start_em.  Var_fun  There are four types of variance specifications; var_fun = 1, the same diagonal variance specification to all K components of the mixture; var_fun = 2, different diagonal variance matrices for different components. var_fun = 3, the same full (unrestricted) variance for all components. var_fun = 4, different full (unrestricted) variance matrices for different components. The default is var_fun	num_runs	Number of function iteration runs, the default is num_runs = 10.
The default is option = 1. More details can be found in start_em.  var_fun  There are four types of variance specifications; var_fun = 1, the same diagonal variance specification to all K components of the mixture; var_fun = 2, different diagonal variance matrices for different components. var_fun = 3, the same full (unrestricted) variance for all components. var_fun = 4, different full (unrestricted) variance matrices for different components. The default is var_fun	start	
variance specification to all K components of the mixture; var_fun = 2, different diagonal variance matrices for different components. var_fun = 3, the same full (unrestricted) variance for all components. var_fun = 4, different full (unrestricted) variance matrices for different components. The default is var_fun	option	
	var_fun	variance specification to all K components of the mixture; var_fun = 2, different diagonal variance matrices for different components. var_fun = 3, the same full (unrestricted) variance for all components. var_fun = 4, different full (unrestricted) variance matrices for different components. The default is var_fun

## Value

The best estimated result (with the smallest AIC value) in the model (Zhang and Einbeck, 2024)  $x_i = \alpha + \beta z_k + \Gamma v_i + \varepsilon_i$  obtained through the EM algorithm.

р	The estimates for the parameter $\pi_k$ , which is a vector of length $K$ .
alpha	The estimates for the parameter $\alpha$ , which is a vector of length $m$ .
z	The estimates for the parameter $z_k$ , which is a vector of length $K$ .
beta	The estimates for the parameter $\beta$ , which is a vector of length $m$ .
gamma	The estimates for the parameter $\Gamma$ , which is a matrix.

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sigma

The estimates for the parameter  $\Sigma_k$ . When var\_fun = 1,  $\Sigma_k$  is a diagonal matrix and  $\Sigma_k = \Sigma$ , and we obtain a vector of the diagonal elements; When var\_fun = 2,  $\Sigma_k$  is a diagonal matrix, and we obtain K vectors of the diagonal elements; When var\_fun = 3,  $\Sigma_k$  is a full variance-covariance matrix,  $\Sigma_k = \Sigma$ , and we obtain a matrix  $\Sigma$ ; When var\_fun = 4,  $\Sigma_k$  is a full variance-covariance matrix, and we obtain K different matrices  $\Sigma_k$ .

W

The posterior probability matrix.

loglikelihood

The approximated log-likelihood of the fitted model.

disparity

The disparity (-2logL) of the fitted model.

number\_parameters

The number of parameters estimated in the EM algorithm.

AIC

The AIC value (-2logL + 2number\_parameters).

BIC

The BIC value  $(-2\log L + number\_parameters*log(n))$ , where n is the number of observations.

aic\_data

All AIC values in each run.

Starting\_values

Lists of starting values for parameters used in each num\_runs. It allows reproduction of the best result (obtained from mult.reg\_1level) in a single run using mult.em\_1level by setting start equal to the list of starting values that were used to obtain the best result in mult.reg\_1level.

#### References

Zhang, Y. and Einbeck J. (2024). A Versatile Model for Clustered and Highly Correlated Multivariate Data. J Stat Theory Pract 18(5).doi:10.1007/s42519023003570

## See Also

```
mult.em_1level.
```

```
##run the mult.em_1level() multiple times and select the best results with the smallest AIC value
set.seed(7)
results <- mult.reg_1level(fetal_covid_data[,c(1:5)],v=fetal_covid_data$status_bi,
K=3, num_runs = 5,steps = 20, var_fun = 2, option = 1)
##Reproduce the best result: the best result is the 5th run in the above example.
rep_best_result <- mult.em_1level(fetal_covid_data[,c(1:5)],
v=fetal_covid_data$status_bi,
K=3, steps = 20, var_fun = 2, option = 1,
start = results$Starting_values[[5]])</pre>
```

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mult.reg_2level	Selecting the best results for multivariate two level model
<b>G</b> =	V

# Description

This wrapper function runs multiple times the function mult.em\_2level for fitting Zhang et al.'s (2023) multivariate response models with two-level random effect, and select the best results with the smallest AIC value.

## Arguments

data	A data set object; we denote the dimension of a data set to be $m$ .
V	Covariate(s).
K	Number of mixture components, the default is K = 2.
steps	Number of iterations within each num_runs, the default is steps = 20.
num_runs	Number of function iteration runs, the default is num_runs = 20.
start	Containing parameters involved in the proposed model (p, alpha, z, beta, sigma, gamma) in a list, the starting values can be obtained through the use of start_em. More details can be found in start_em.
option	Four options for selecting the starting values for the parameters in the model. The default is option = 1. More details can be found in start_em.
var_fun	There are two types of variance specifications; var_fun = 1, the same diagonal variance specification to all K components of the mixture; var_fun = 2, different diagonal variance matrices for different components; The default is var_fun = 2.

## Value

The best estimated result (with the smallest AIC value) in the model  $x_{ij} = \alpha + \beta z_k + \Gamma v_{ij} + \varepsilon_{ij}$  obtained through the EM algorithm (Zhang et al., 2023), where the upper-level unit is indexed by i, and the lower-level unit is indexed by j.

p	The estimates for the parameter $\pi_k$ , which is a vector of length $K$ .
alpha	The estimates for the parameter $\alpha$ , which is a vector of length $m$ .
Z	The estimates for the parameter $z_k$ , which is a vector of length $K$ .
beta	The estimates for the parameter $\beta$ , which is a vector of length $m$ .
gamma	The estimates for the parameter $\Gamma$ , which is a matrix.
sigma	The estimates for the parameter $\Sigma_k$ . When var_fun = 1, $\Sigma_k$ is a diagonal matrix and $\Sigma_k = \Sigma$ , and we obtain a vector of the diagonal elements; When var_fun = 2, $\Sigma_k$ is a diagonal matrix, and we obtain K vectors of the diagonal elements.
W	The posterior probability matrix.
loglikelihood	The approximated log-likelihood of the fitted model.
disparity	The disparity (-2logL) of the fitted model.

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number\_parameters

The number of parameters estimated in the EM algorithm.

AIC The AIC value (-2logL + 2number\_parameters).

aic\_data All AIC values in each run.

Starting\_values

Lists of starting values for parameters used in each num\_runs. It allows reproduction of the best result (obtained from mult.reg\_2level) in a single run using mult.em\_2level by setting start equal to the list of starting values that were used to obtain the best result in mult.reg\_2level.

#### References

Zhang, Y., Einbeck, J. and Drikvandi, R. (2023). A multilevel multivariate response model for data with latent structures. In: Proceedings of the 37th International Workshop on Statistical Modelling, pages 343-348. Link on RG: https://www.researchgate.net/publication/375641972\_A\_multilevel\_multivariate\_response\_model\_for\_data\_with\_latent\_structures

#### See Also

```
mult.em_2level.
```

## **Examples**

start\_em

Starting values for parameters

### **Description**

The starting values for parameters used for the EM algorithm in the functions: mult.em\_1level, mult.em\_2level, mult.reg\_1level and mult.reg\_2level.

## **Arguments**

data A data set object; we denote the dimension of a data set to be m.

v Covariate(s); we denote the dimension of it to be r. K Number of mixture components, the default is K = 2.

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steps

Number of iterations. This will only be used when using option = 2 for both the 1-level model and the 2-level model. It should also be used when using option = 3 and option = 4 for the 1-level model, provided var\_fun is set to either 3 or 4; the default is steps = 20.

option

Four options for selecting the starting values for the parameters. The default is option = 1. When option = 1:  $\pi_k = \frac{1}{K}$ ,  $z_k \sim \text{rnorm}(K, \text{mean} = 0, \text{sd=1})$ ,  $\alpha$  = column means,  $\beta$  = a random row minus alpha,  $\Gamma$  = coefficient estimates from separate linear models,  $\Sigma$  is diagonal matrix where the diagonals take the value of column standard deviations over K; when option = 2: use a short run (steps = 5) of the EM function which uses option = 1 with var\_fun = 1 and use the estimates as the starting values for all the parameters; when option = 3: the starting value of  $\beta$  is the first principal component, and the starting values for the rest of the parameters are the same as described when option = 1; when option = 4: first, take the scores of the first principal component of the data and perform K-means,  $\pi_k$  is the proportion of the clustering assignments, and  $z_k$ take the values of the K-means centers, and the starting values for the rest of the parameters are the same as described when option = 1.

var\_fun

The four variance specifications. When var\_fun = 1, the same diagonal variance specification to all K components of the mixture; var\_fun = 2, different diagonal variance matrices for different components. var\_fun = 3, the same full (unrestricted) variance for all components. var\_fun = 4, different full (unrestricted) variance matrices for different components. If unspecified, var\_fun = 2. Note that for application propose, in two-level models, var\_fun can only take values of 1 or 2.

р

optional; specifies starting values for  $\pi_k$ , it is input as a K-dimensional vector. optional; specifies starting values for  $z_k$ , it is input as a K-dimensional vector.

Z

optional; specifies starting values for  $\beta$ , it is input as an m-dimensional vector.

beta alpha

optional; specifies starting values for  $\alpha$ , it is input as an m-dimensional vector.

sigma

optional; specifies starting values for  $\Sigma_k$  ( $\Sigma$ , when var\_fun = 1 or var\_fun = 3), when  $var_fun = 1$ , it is input as an m-dimensional vector, when  $var_fun = 1$ 2, it is input as a list (of length K) of m-dimensional vectors, when var\_fun = 3, it is input as an  $m \times m$  matrix, when  $var_fun$  = 4, it is input as a list (of

length K) of  $m \times m$  matrices.

gamma

optional; the coefficients for the covariates; specifies starting values for  $\Gamma$ , it is input as an  $m \times r$  matrix.

#### Value

The starting values (in a list) for parameters in the models  $x_i = \alpha + \beta z_k + \Gamma v_i + \varepsilon_i$  (Zhang and Einbeck, 2024) and  $x_{ij} = \alpha + \beta z_k + \Gamma v_{ij} + \varepsilon_{ij}$  (Zhang et al., 2023) used in the four functions: mult.em\_1level, mult.em\_2level, mult.reg\_1level and mult.reg\_2level.

p	The starting value for the parameter $\pi_k$ , which is a vector of length $K$ .
alpha	The starting value for the parameter $\alpha$ , which is a vector of length $m$ .
z	The starting value for the parameter $z_k$ , which is a vector of length $K$ .
beta	The starting value for the parameter $\beta$ , which is a vector of length $m$ .

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gamma

The starting value for the parameter  $\Gamma$ , which is a matrix.

sigma

The starting value for the parameter  $\Sigma_k$ . When  $\text{var\_fun} = 1$ ,  $\Sigma_k$  is a diagonal matrix and  $\Sigma_k = \Sigma$ , and we obtain a vector of the diagonal elements; When  $\text{var\_fun} = 2$ ,  $\Sigma_k$  is a diagonal matrix, and we obtain K vectors of the diagonal elements; When  $\text{var\_fun} = 3$ ,  $\Sigma_k$  is a full variance-covariance matrix,  $\Sigma_k = \Sigma$ , and we obtain a matrix  $\Sigma$ ; When  $\text{var\_fun} = 4$ ,  $\Sigma_k$  is a full variance-covariance matrix, and we obtain K different matrices  $\Sigma_k$ .

#### References

Zhang, Y., Einbeck, J. and Drikvandi, R. (2023). A multilevel multivariate response model for data with latent structures. In: Proceedings of the 37th International Workshop on Statistical Modelling, pages 343-348. Link on RG: https://www.researchgate.net/publication/375641972\_A\_multilevel\_multivariate\_response\_model\_for\_data\_with\_latent\_structures.

Zhang, Y. and Einbeck, J. (2024). A Versatile Model for Clustered and Highly Correlated Multivariate Data. J Stat Theory Pract 18(5).doi:10.1007/s42519023003570

## **Examples**

```
##example for the faithful data.
data(faithful)
start <- start_em(faithful, option = 1)</pre>
```

trading\_data

A set of import and export data in 44 countries.

#### Description

The variables are given as the percentage of imports and exports in relation to the overall GDP. The data set comprises data from 44 countries (for our analysis), we specifically selected the time period between 2018 and 2022.

#### Usage

```
data(trading_data)
```

#### **Format**

An object of class "data.frame"

**import** The fetus from the same twins share the same id number.

**export** frequency of self-touch for each fetus.

country frequency of twin-to-twin for each fetus.

#### Source

Trade in Goods and Services. https://data.oecd.org/trade/trade-in-goods-and-services.htm. Accessed on 2023-05-29.

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#### **Examples**

```
data(trading_data)
head(trading_data)
```

twins\_data

A set of fetal movements data in twins and singletons.

#### **Description**

This data was collected for research on the effects of maternal mental health on prenatal movements in twins and singletons (Reissland et al., 2021). There are two touch movement types of the fetus recorded: self-touch and twin-to-twin touch, and the mothers' mental health status was collected on three variables: depression, perceived stress scale and stress. There are 14 pairs of twins, 11 of the mothers were available for one scan and 3 of them were available for two scans, i.e. in total there are 34 observations.

## Usage

```
data(twins_data)
```

#### **Format**

An object of class "data.frame"

id The fetus from the same twins share the same id number.

SelfTouchCodable frequency of self-touch for each fetus.

OtherTouchCodable frequency of twin-to-twin for each fetus.

**Depression** Depression scale of the mothers.

**PSS** Perceived Stress Scale of the mothers.

Anxiety Hospital Anxiety of the mothers.

## References

Reissland, N., Einbeck, J., Wood, R., and Lane, A. (2021). Effects of maternal mental health on prenatal movement profiles in twins and singletons. Acta Paediatrica, 110(9):2553–2558.

```
data(twins_data)
head(twins_data)
```

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